Game Theory in Wireless Communications with an Application to Signal Synchronization

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***Abstract*—This paper is concerned with the general issue of game-theoretic techniques applied to the problem of resource allocation in wireless communication networks. Specifically, its first part is devoted to a tutorial explanation of game theory in the context of CDMA wireless networking, whilst the second part focuses on the particular issue of allocating power resources to optimize the receiver performance in terms of spreading code acquisition. The problem of initial signal acquisition is formulated as a noncooperative game in which each transmitter-receiver pair in the network seeks to maximize a specifically chosen utility function. For the problem at hand, the most significant utility function is represented by the ratio of the probability of signal detection to the transmitted energy per bit, and the game each receiver plays consists in setting its own transmit power and detection threshold, under a constraint on the maximum probability of spurious code locks. This formulation of the game captures the tradeoff between obtaining good code acquisition performance and saving as much energy as possible. Using the techniques introduced with the "toy examples" in the first part of the paper, the Nash solution of the proposed game is investigated and found. Closed-form expressions for the optimal transmit power and detection threshold at the Nash equilibrium are derived, and they are compared with simulation results for a decentralized resource control algorithm.**

***Index Terms*—Game theory, Nash equilibrium, resource allocation, power control, multiuser wireless networks, CDMA, synchronization, code acquisition.**

I. Introduction

As is known, *game theory* is a broad field of applied mathematics that aims at describing and analyzing interactive decision processes. It in fact provides analytical tools to predict the outcome of complex interactions among rational entities, where rationality calls for strict adherence to a strategy based on perceived or measured results [1]. Traditionally, the main areas of its application have been economics, political science, biology, and sociology. But, since the early 1990s, engineering and computer science have been added to the list.

Wireless communications is a suitable scenario for the application of game theory. Since the early days of wireless communications, the importance of radio resource management (RRM) has in fact emerged as a key issue in network design. Cochannel interference, which is due to the shared nature of the wireless medium, represents a major impairment to the performance of wireless communications. The resource competition can be investigated by modeling the network as an economic system, in which any action taken by a user affects the performance of others as well: just the main field of application of game theory. For instance, there is a substantial literature on power control techniques based on noncooperative game theory, mostly focused on data detection for code division multiple access (CDMA) wireless communications networks (e.g., [2]–[26]). The first part of this paper is devoted to a review of game theory and its applications for power allocation, followed by a simple ("toy") case study of power control for CDMA, that is expedient to explain the main concepts just reviewed (players, utility, Nash equilibrium, Pareto optimality, etc.)

On the other hand, in addition to data detection, every uplink receiver in the base station of a CDMA network must perform the fundamental function of initial signal synchronization to lock onto the terminal's signature code. The conjecture we try to (dis)prove here is that the optimum decentralized power allocation devised for data detection leads also to a resource allocation strategy that is good for such synchronization function. To the best of our knowledge, similar approaches focusing on synchronization performance have not been investigated in the literature. In particular, we will develop here a game-theoretic analysis to address the problem of optimal resource allocation in a CDMA wireless network so as to improve the performance of code synchronization. Due to the multiple access interference (MAI) caused by the concurrent presence of terminals sharing the same medium, the performance in terms of code synchronization accuracy is affected by the transmission parameters of all users in the network (e.g., transmit power, modulation format, spreading factor). We will introduce and analyze a noncooperative (distributed) game in which the users in the network are allowed to tune their terminals (at both the transmitter and receiver side) to maximize the ratio between the probability of detection to the transmitted energy per bit.

The remainder of the paper is structured as follows. Section II introduces the basics of noncooperative game theory by means of some expedient examples, and sets the basis for this work. The system model is given in Section III, whereas the formulation of the problem and the Nash equilibrium of the noncooperative game are described in Section IV. Section V shows some numerical results for the proposed analysis, and some conclusions and perspectives are outlined in Section VI.

II. Game-theoretic formulation of CDMA power control

After the seminal contributions to noncooperative game theory given in the 50's by J. F. Nash (in particular the proof of the existence of a strategic equilibrium for noncooperative

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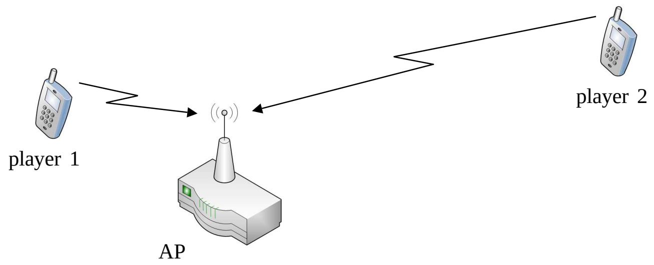

Fig. 1. The network scenario in the near-far effect game.

near player (player 1) \ far player (player 2)	0	p
0	$0, 0$	$0, 1-c$
p	$1-c, 0$	$1-c, -c$

$u_1(p_1, p_2), u_2(p_1, p_2)$

Fig. 2. Payoff matrix for the near-far effect game.

players - the so-called the *Nash equilibrium* [27], [28]), game theory has been a subject of considerable study, especially in the traditional areas of economics and political science.

More recently, game theory has been also used in telecommunications and wireless communications [29]–[36]. The reason for this blooming of game-theoretic applications lies in the nature of typical interactions between users in a wireless network. The wireless terminals can in fact be modeled as players in a game competing for the network resources (e.g., bandwidth and power), which are typically scarce. Any action taken by a user affects the performance of other users as well. Thus, game theory turns out to be a natural tool for investigating this interplay.

In the next subsections, we will provide some motivating examples for its use in the context of power control for multiaccess wireless networks, also introducing some key definitions and the fundamental analytical tools.

A. Noncooperative games

A (strategic) game consists of three components: a set of players, the strategy set for each player, and a utility (payoff) for each player measuring its level of satisfaction [37]. In its mathematical formulation, the game can be represented as $\mathcal{G} = [\mathcal{K}, \{\mathcal{A}_k\}, \{u_k(\mathbf{a})\}]$, where $\mathcal{K} = \{1, \ldots, K\}$ is the set of players; $\mathcal{A}_k$ is the set of actions (strategies) available to player k; and $u_k(\mathbf{a})$ is the utility (payoff) for player k. The utility depends not only on its own strategy $a_k \in \mathcal{A}_k$, but also on the actual strategies taken by *all* of the other players, denoted by $\mathbf{a}_{\setminus k} = (a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_K)$. Hence, $u_k(\mathbf{a}) = u_k(a_k, \mathbf{a}_{\setminus k})$.

Since our focus is on distributed schemes, we concentrate on *noncooperative games*, where each player k chooses its strategy a_k^* to *unilaterally* maximize its own utility $u_k(\mathbf{a})$:

$$\begin{aligned} a_k^* &= \arg\max_{a_k \in \mathcal{A}_k} u_k(\mathbf{a}) \\ &= \arg\max_{a_k \in \mathcal{A}_k} u_k(a_k, \mathbf{a}_{\setminus k}), \end{aligned} \tag{1}$$

where the latter notation emphasizes that the k-th player has control over its *own* strategy a_k only. In other terms, a_k^* represents player k's *best response* to the concurrent actions of the other players. For these games, we first need to introduce two fundamental concepts, namely, *Nash equilibrium* and *Pareto optimality* [37].

Definition 1: A *Nash equilibrium* (NE) is a set of strategies $\mathbf{a}^* = (a_k^*, \mathbf{a}_{\setminus k}^*)$ such that no user can unilaterally improve its own utility, i.e.,

$$u_k(a_k^*, \mathbf{a}_{\setminus k}^*) \geq u_k(a_k, \mathbf{a}_{\setminus k}^*) \quad \forall a_k \in \mathcal{A}_k, k \in \mathcal{K}, \tag{2}$$

where obviously $\mathbf{a}_{\setminus k}^* = (a_1^*, \ldots, a_{k-1}^*, a_{k+1}^*, \ldots, a_K^*)$. In other words, an NE is a stable outcome of a game in which multiple agents with conflicting interests compete through self-optimization and reach a point where no player has any incentive to *unilaterally* deviate (whence stability).

Definition 2: A set of strategies $\tilde{\mathbf{a}} = (\tilde{a}_k, \tilde{\mathbf{a}}_{\setminus k})$ is *Pareto-optimal* if there exists no other set $\mathbf{a} = (a_k, \mathbf{a}_{\setminus k})$ such that $u_k(a_k, \mathbf{a}_{\setminus k}) \geq u_k(\tilde{a}_k, \tilde{\mathbf{a}}_{\setminus k})$ for all $k \in \mathcal{K}$ and $u_k(a_k, \mathbf{a}_{\setminus k}) > u_k(\tilde{a}_k, \tilde{\mathbf{a}}_{\setminus k})$ for some $k \in \mathcal{K}$. In other words, when all players settle onto a Pareto-optimal strategy, no player can improve its own utility without reducing the utility of at least another player.

Our focus throughout this work is on *pure* (i.e., deterministic) strategies. However, players can also opt for *mixed* (i.e., statistical) strategies. In this case, each player chooses its strategy according to a *probability distribution* that is known to the other players. Nash proved that a finite noncooperative game always has at least one mixed-strategy NE [27], [28]. This means that a noncooperative game may have no pure-strategy equilibria, one pure-strategy equilibrium, or multiple pure-strategy equilibria. We can also easily show that there could exist more than one Pareto-optimal solution, and that, in general, an NE does not correspond to a Pareto-optimal strategy.

To illustrate the intuitive meaning of these concepts, we consider a trivial example of a static[1] noncooperative game, that we call the *near-far effect game*. Two wireless terminals (player 1 and player 2) transmit to a certain access point (AP) in a CDMA network. Player 1 is located close to the AP, whilst player 2 is much farther away, as is depicted in Fig. 1. Hence, $K = 2$ and $\mathcal{K} = \{1, 2\}$. Each user is allowed either to transmit at a certain power level $p_k = p$, or to wait ($p_k = 0$). This translates into $\mathcal{A}_k = p_k = \{0, p\}$. Each terminal achieves a degree of satisfaction which depends on the outcome of the transmission and on the expenditure in terms of cost of the

[1] A game is said to be *static* if there exists only one time step, which means that the players' strategies are carried out through a single move [37].

energy spent to transmit at power p_k. Mathematically, this translates into an adimensional utility $u_k(\mathbf{a}) = u_k(p_1, p_2) = t_k - c_k$, where $t_k = 1$ if the transmission is successful and $t_k = 0$ otherwise, and where the cost is $c_k = c \ll 1$ if the player chooses to transmit, and $c_k = 0$ otherwise.

Due to the near-far effect, sketched in Fig. 1, whenever the near player (player 1) chooses to transmit, its transmission is successful irrespective of the action of the far player (player 2). In particular, if $p_1 = p$, player 1 can deliver its information, thus receiving a utility $u_1(p, p_2) = 1 - c$ (irrespective of p_2). When on the contrary player 1 is idle, i.e., $p_1 = 0$, its own utility is $u_1(0, p_2) = 0$, irrespective of p_2 again. Let us now focus on player 2. Because of the interference caused by player 1, player 2 can only successfully transmit when player 1 is idle ($p_1 = 0$). In this case, $u_2(0, p) = 1 - c$. If both players transmit $p_k = p$, due to the near-far effect, player 2's transmission fails and $u_2(p, p) = -c$. Similarly to player 1, $u_2(p_1, 0) = 0$ (irrespective of player 1) when player 2 is idle. The near-far effect game above is summarized in the *payoff matrix* of Fig. 2. Player 1's actions are identified by the rows and player 2's by the columns. The pair of numbers in the box represents the utilities $(u_1(p_1, p_2), u_2(p_1, p_2))$ achieved by the players.

To predict the outcome of the near-far effect game, it is fundamental to assume that both players i) are rational, and ii) know each other's payoff.[2] By inspecting the payoff matrix, it is apparent that player 1's best strategy is represented by $p_1 = p$ whatever p_2 is, since $1 - c > 0$ under the assumption $c \ll 1$. This is known to player 2 as well. Hence, to "limit damage", he/she rationally chooses to play $p_2 = 0$. As a conclusion, the near-far effect game has only one pure-strategy NE, represented by the strategy $\mathbf{a} = (p, 0)$ (the same conclusion follows from Definition 1).

By applying Definition 2, this game can be shown to have two Pareto-optimal solutions, namely $(p, 0)$ and $(0, p)$, corresponding to transmission of player 1 only or player 2 only, respectively. It is true that the (only) pure-strategy NE solution $(p, 0)$ is also Pareto-optimal, but it is also true that i) it is highly unsatisfactory for player 2 since he/she can not convey any information to the AP, and that ii) the other Pareto-optimal solution can *not* be attained by a noncooperative game (since it is not a NE). We take this apparent need for *fairness* as our motivation to introduce power control.

B. Power control as a static noncooperative game

Let us provide our near-far effect game with a naive form of *power control*. Assume now that each terminal is allowed to transmit choosing between two levels of transmit power different from the previous ones, namely, either at a certain amount p, or at a reduced level μp, $0 < \mu < 1$. The power control factor μ is such that the received power for both players is the same when the far player uses p and the near player uses μp. Hence, $\mathcal{A}_k = p_k = \{\mu p, p\}$. Similarly to the previous game with no power control, $u_k(p_1, p_2) = t_k - c_k$, where $t_k = 1$ if the transmission for player k is successful, and $t_k = 0$ otherwise, and where c_k is proportional to the consumed energy, i.e., $c_k = c$ if $p_k = p$, and $c_k = \mu c$ if $p_k = \mu p$. As before, due to the near-far effect, player 1 can successfully transmit irrespective of p_2, whereas player 2 can correctly reach the receiver only if $p_2 > p_1$. The payoff matrix for this game is shown in Fig. 3. Since $1 - \mu c > 1 - c$, player 1's best strategy is $p_1 = \mu p$. Consequently, player 2 plays $p_2 = p$. This game has thus one pure-strategy NE, which is also the only Pareto-optimal solution, and player 2 now succeeds to go through when player 1 is idle.

		far player (player 2)	
		μp	p
near player (player 1)	μp	$1 - \mu c, -\mu c$	$1 - \mu c, 1 - c$
	p	$1 - c, -\mu c$	$1 - c, -c$

$u_1(p_1, p_2), u_2(p_1, p_2)$

Fig. 3. Payoff matrix for the near-far effect game with power control and zero-one utility.

This power control technique seems to compensate for the near-far effect, since both players are now able to transmit, although player 1 still tends to dominate player 2. However, this scenario does not actually model real data networks. The main inaccuracy lies in the over-simplified "go/no-go" utility function that does not take into account the actual signal-to-interference-plus-noise noise ratio (SINR) achieved at the receiver. In a data network, higher SINRs lead to a larger amount of transmitted information. This implies that the utility for a data terminal is a continuous function of its achieved SINR.

To account for this different point of view, the term t_k should be a function of the amount of information that is actually delivered to the receiver. Focusing on player 1, if $p_1 = p$, the (normalized) amount of information (we may call it the *throughput*) is equal to $t_1 = t \gg c$. If player 1 uses a lower power $p_1 = \mu p$, then $t_1 = \lambda t$, with $\mu < \lambda \lesssim 1$.[3] Considering player 2, $t_2 = 0$ if $p_2 \le p_1$, and $t_2 = \lambda t$ if $p_2 > p_1$, since the received power for player 2 is equal to that of player 1 with $p_1 = \mu p$.

The payoff matrix for this more realistic game is now shown in Fig. 4. As before, player 1's best strategy is represented by $p_1 = p$ whatever p_2 is, since $t - c > \lambda t - \mu c$ under the assumption $t \gg c$. As a consequence, player 2 rationally chooses to play $p_2 = \mu p$. The pure-strategy NE is represented by the strategy $(p, \mu p)$, whereas the Pareto-optimal solutions are $(p, \mu p)$ and $(\mu p, p)$. We appear to be back to the original situation we had without power control. However, we can make this situation considerably fairer by introducing a new

[2]This hypothesis involves the concept of *complete information* [37].

[3]Note that $\mu < \lambda$ in all practical scenarios, since the performance in terms of correct detection does not show a linear dependence on the transmit power.

near player (player 1) \ **far player (player 2)**	μp	p
μp	$\lambda t - \mu c, -\mu c$	$\lambda t - \mu c, \lambda t - c$
p	$t - c, -\mu c$	$t - c, -c$

$u_1(p_1, p_2), u_2(p_1, p_2)$

Fig. 4. Payoff matrix for the near-far effect game with power control and variable throughput.

concept and a new kind of game. First, we introduce the notion of *social optimality*: a solution of our game is *socially optimal* if the overall utility $u_{\text{net}}(p_1, p_2) = u_1(p_1, p_2) + u_2(p_1, p_2)$ is maximal. We see that $(p, \mu p)$, in spite of being Pareto-optimal, does *not* represents the socially optimal solution for the network as a whole, since the overall utility is $u_{\text{net}}(p, \mu p) = t - (1 + \mu) c < 2\lambda t - (1 + \mu) c = u_{\text{net}}(\mu p, p)$. Hence, the NE is not desirable in a social sense.

How can we attain the Pareto optimal solution with a decentralized strategy, then? The answer is the *dynamic game* [37], i.e., a game involving the repetition of many moves of the same static game. It is easy to see that $(p, \mu p)$ is the best strategy for player 1 in a one-move (static) game only. If the near-far effect game of Fig. 4 is played with several moves, player 1 will choose the strategy $p_1 = \mu p$. To see the motivation for this, assume that $1/\lambda$ is an integer for the sake of simplicity (the same conclusion holds even when such assumption does not hold). If the players play $(\mu p, p)$ for $1/\lambda$ times, player 1 achieves a total utility $(\lambda t - \mu c)/\lambda = t - \mu c/\lambda$, which is greater than $t - c$ since $\mu < \lambda$. The only disadvantage is an increased transmission time, which is not necessarily a negative feature (it actually is only for delay-sensitive applications, which call for different utility functions).

C. Power control as a dynamic noncooperative game

The toy examples described above are expedient to understand how game theory can be applied to derive distributed power control techniques for wireless data networks. In this section, we will review the standard approach based on game theory to derive a power control scheme for multiuser data networks focusing on the issue of *data detection*. From the discussions above, we have seen that scalable resource allocation techniques can be modeled as dynamic noncooperative games. We also noticed that the utility function has a dramatic impact on the nature of the game. Since wireless data networks are often populated by many battery-powered mobile terminals, a primary goal is the maximization of the number of transmitted bits per energy unit rather than the pure maximization of the throughput of the link. This goal can be achieved through application of a noncooperative game wherein the users are allowed to choose their transmit powers according to a utility-maximization criterion, where the *utility* is defined as the ratio of throughput to transmit power. Although the focus of the game described in the following is on data detection, the *energy-efficient* paradigm also applies to the game-theoretic formulation derived in the next sections and tailored to the problem of code synchronization for multiaccess wireless data networks.

Let $\mathcal{G} = [\mathcal{K}, \{\mathcal{P}_k\}, \{u_k(\mathbf{p})\}]$ be the dynamic noncooperative power control game, where $\mathcal{K} = \{1, \dots, K\}$ is the index set for the terminal users of the multiuser wireless network; $\mathcal{P}_k = [0, \overline{p}_k]$ is the strategy set; and $u_k(\mathbf{p})$ is the payoff function for user k. Note that, unlike previous examples, here the allowed transmit powers can take all the values in the *continuous* interval $[0, \overline{p}_k]$.

In terms of the utility function, several different definitions were investigated in the literature, depending on the goal to be achieved [3], [10], [13]–[15]. When energy efficiency is the main concern, as it is here, there exists a tradeoff between obtaining high SINR levels and consuming low energy. A good way to assess this tradeoff is through the number of bits that can be transmitted per joule of energy consumed [2], [4]. This can be quantified by defining the utility function of the kth user to be the ratio of its *throughput* T_k to its *transmit power* p_k, i.e.,

$$u_k(\mathbf{p}) = u_k\left(p_k, \mathbf{p}_{\setminus k}\right) = \frac{T_k}{p_k}, \tag{3}$$

where $\mathbf{p} = (p_1, \dots, p_K) = (p_k, \mathbf{p}_{\setminus k})$ represents the vector of transmit powers, with K denoting the number of users, and $\mathbf{p}_{\setminus k}$ representing the vector of elements of $\mathbf{p}$ other than the kth element. The throughput T_k (sometimes referred to as *goodput*) can be expressed as

$$T_k = R_k f(\gamma_k), \tag{4}$$

where R_k and γ_k are the transmission rate and the received SINR for the kth user, respectively. The function $f(\gamma_k)$ is known as the *efficiency function*, which expresses the packet success rate (PSR), i.e., the probability that a packet is received without an error, as a function of SINR. We assume that a packet is retransmitted if it has one or more bit errors, i.e., no forward error correction (FEC) techniques are considered. Of course, $f(\gamma_k)$ depends on the details of the physical layer, including modulation, coding, and packet size. However, in most practical cases, the efficiency function shows some common features: $f(\gamma_k)$ is increasing, S-shaped (sigmoidal), continuously differentiable, with $f(0) = 0$, $f(+\infty) = 1$, and $f'(0) = df(\gamma_k)/d\gamma_k|_{\gamma_k=0} = 0$ [38]. Combining (3) and (4), we can write

$$u_k(\mathbf{p}) = R_k \frac{f(\gamma_k)}{p_k}. \tag{5}$$

This utility function, which has units of bits/joule, represents the total number of correct data bits that are delivered to the destination, and captures the tradeoff between data rate and battery life. Fig. 5 shows a typical shape of the utility function in (5) as a function of transmit power of user k keeping all of the other users' transmit power constant. The

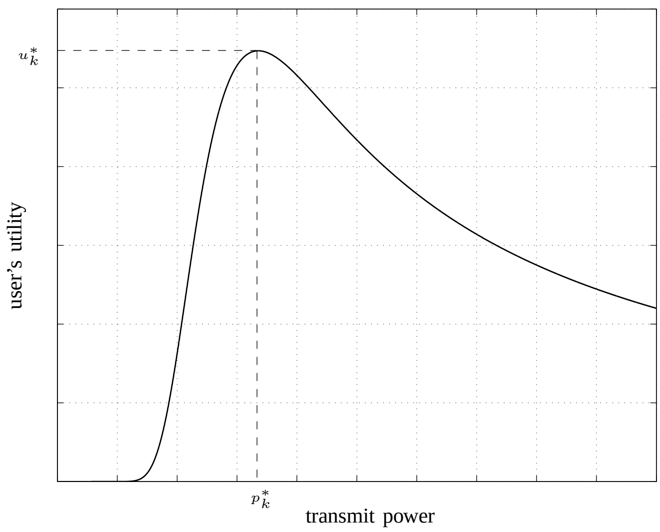

Fig. 5. User's utility as a function of transmit power for a fixed interference.

utility function in (5) can also be used for coded systems by modifying the efficiency function $f(\gamma_k)$ to represent the PSR for coded systems and also scaling the transmission rate to count only the information bits in a packet.

Recalling (1), and considering $\mathcal{P}_k = [0, \overline{p}_k]$, player k's best response $r_k(\mathbf{p}_{\setminus k})$ to a given interference vector $\mathbf{p}_{\setminus k}$ is [5]

$$r_k(\mathbf{p}_{\setminus k}) = \min(\overline{p}_k, p_k^*), \tag{6}$$

where

$$\begin{aligned} p_k^* &= \arg\max_{p_k \in \mathbb{R}^+} u_k(p_k, \mathbf{p}_{\setminus k}) \\ &= \arg\max_{p_k \in \mathbb{R}^+} R_k \frac{f(\gamma_k)}{p_k} \end{aligned} \tag{7}$$

is the unconstrained maximizer of the utility in (5). Fig. 5 illustrates the quantities p_k^* and $r_k(\mathbf{p}_{\setminus k})$. Recall that terminal k has control over its own power p_k only, and that its action is chosen to maximize its own utility. The value p_k^* represents the transmit power that guarantees the best payoff u_k^* given a set of interferers $\mathbf{p}_{\setminus k}$. Clearly, if this value is not feasible, i.e., if $p_k^* > \overline{p}_k$, then the best action the user k can choose is transmitting at the maximum value $\overline{p}_k$.

We can show [38] that in a CDMA network over a flat-fading wireless channel, u_k^* occurs when the terminal k, using the transmit power p_k^*, achieves the optimal (target) SINR γ_k^*, which is a function of its own transmit parameters only. As a consequence, the target SINR γ_k^* can be computed by each player k before the game starts. Using the analysis developed in [5], (7) can be used to provide an *iterative algorithm*, which mimics a *dynamic game* to guarantee a good tradeoff between fairness of the network and efficiency of the resource allocation scheme. At the $(n+1)$-th step, user k updates its (optimal) transmit power $p_k^*(n+1)$ according to

$$p_k^*(n+1) = p_k^*(n) \cdot \frac{\gamma_k^*}{\gamma_k(n)}, \tag{8}$$

where $p_k^*(n)$ is the transmit power at the n-th step, and $\gamma_k(n)$ is the SINR experienced by user k at the n-th step, that can be fed back by the AP using a return channel. Note that the update process (8) recalls the SINR-balancing criterion derived in the milestone works in the field of distributed power control [39]–[42]. Using the analysis presented in [5], we can show that the Nash equilibrium (7) exists and is unique, although it is not Pareto efficient. Methods to move the Nash equilibrium closer to the Pareto-optimal solution make use of pricing techniques, and can be found in [4], [5]. This algorithm can be extended to the case of frequency-selective scenarios using the analysis presented in [11], [12].

III. Game-Theoretic Power Control for Signal Acquisition

The examples described in the previous section are primarily focused on the issue of data detection for CDMA wireless networks. In the following, we will present a game-theoretic resource allocation scheme based on energy efficiency, but specifically suited to the problem of *code synchronization*. In this context, the maximization of the achieved throughput per energy consumed is replaced by the maximization of the probability of code acquisition per energy consumed.

In the uplink of a multiaccess CDMA infrastructure network, we consider code acquisition on a pilot channel (i.e., either with no data modulation or modulated with known data), and we assume for simplicity the presence of K equi-format users with binary signaling and a common spreading factor M (i.e., the transmission bit rate $R_b = 1/T_b$ is common among all users, where $T_b = MT_c$ is the bit time and T_c is the chip time). The transmission takes place over a frequency-flat and slow-fading additive white Gaussian noise (AWGN) channel. Using baseband-equivalent representation, the signal transmitted by each user l can be expressed as

$$s_l(t) = \sqrt{2p_l} \sum_n g_l(t - nT_b)$$

where p_l is the l-th user's transmit power, and where

$$g_l(t) = \sum_{m=0}^{M-1} c_m^{(l)} \alpha(t - mT_c) \tag{9}$$

is the l-th user's signature (bandlimited) waveform. In (9), $\mathbf{c}_l = \{c_m^{(l)}\}_{m=0}^{M-1}$ denotes the spreading code for user l, which is assumed to be random, with $c_m^{(l)} \in \{\pm 1\}$, and

$$\mathbb{E}\left\{c_m^{(l)} \cdot c_{m+\ell}^{(l)}\right\} = \begin{cases} 1, & \ell = 0, \\ 0, & \ell \neq 0 \end{cases} \tag{10}$$

$$\mathbb{E}\left\{c_m^{(l)} \cdot c_{m+\ell}^{(j)}\right\} = 0, \qquad \forall j \neq l, \qquad \forall \ell, \tag{11}$$

where $\mathbb{E}\{\cdot\}$ denotes statistical expectation. Also, $\alpha(t)$ is a square root raised cosine (SRRC) pulse with energy T_c (the chip shaping pulse). No data modulation is present.

Assuming perfect carrier frequency synchronization, the received signal is

$$r(t) = \sum_{l=1}^{K} h_l e^{j\theta_l} s_l(t - \tau_l) + \eta(t), \tag{12}$$

where h_l, θ_l, and τ_l are the attenuation, the phase offset, and the delay, respectively, experienced by the l-th user's

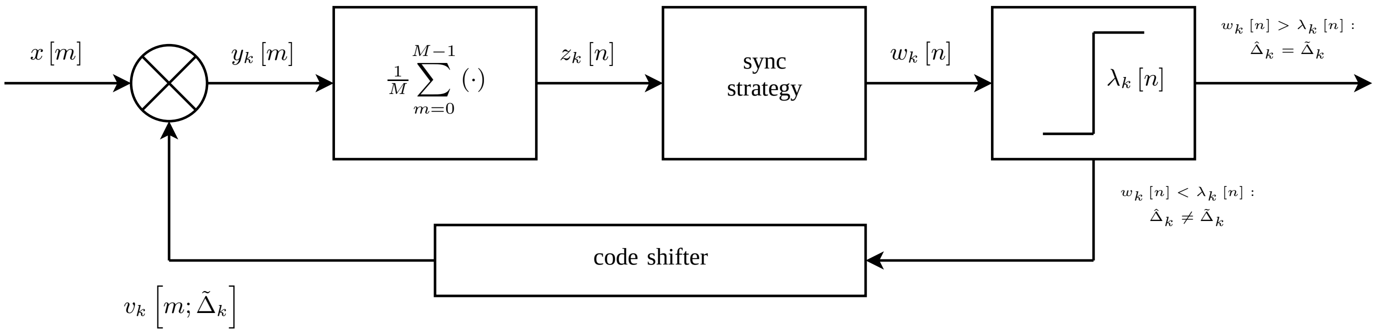

Fig. 6. Serial search architecture for user k's code shift Δ_k.

signal when propagating through the wireless channel; and $\eta(t)$ represents the zero-mean complex-valued AWGN with two-sided power spectral density (PSD) $2N_0$. To simplify our problem, we concentrate on a chip-synchronous scenario, i.e., the unknown signal delay to be estimated is an integer multiple of the chip interval T_c: $\tau_l = \Delta_l T_c$ for every l, where the code shift Δ_l introduced by the channel is uniformly distributed in $\{0, 1, \cdots, M-1\}$. After chip-matched filtering and sampling, the received signal at the uplink receiver can be represented as

$$\begin{aligned} x[m] &= r(t) \otimes \frac{1}{T_c}\alpha(-t)\Big|_{t=mT_c} \\ &= \sum_{l=1}^{K} h_l e^{j\theta_l}\sqrt{2p_l}\, c^{(l)}_{m+\Delta_l} + \nu[m], \end{aligned} \tag{13}$$

where $\nu[m] = \nu_I[m] + j\nu_Q[m]$ is Gaussian-distributed, with independent components $\nu_I[m], \nu_Q[m] \sim \mathcal{N}(0, \sigma^2)$, and $\sigma^2 = N_0/T_c$ is the noise power of each component. This model can be easily extended to totally distributed systems, such as ad-hoc networks, provided that all channel components h_l, θ_l, τ_l, are replaced by their counterparts h_{lk}, θ_{lk}, τ_{lk}, that account for the l-th transmitter / k-th receiver pair.

Coming back to the infrastructure configuration, in order for the base station to lock the spreading codes of all K users in the network, the receiver is equipped with K detectors to search for the correct code shifts Δ_k for all $k \in [1, \cdots, K]$. The simplest technique that can be employed is the well-known *serial search* sketched in Fig. 6 for user k, that is applied since the early days of CDMA [43]–[45]. The sufficient statistics to test code alignment is obtained after despreading and weighing the received samples $x[m]$ as follows:

$$v_k\left[m; \tilde{\Delta}_k\right] = \frac{1}{h_k\sqrt{2p_k}} \cdot c^{(k)}_{m+\tilde{\Delta}_k}$$

where $\tilde{\Delta}_k$ is the tentative code shift of the locally generated sequence $\mathbf{c}_k$. Note that the receiver for user k must estimate the k's transmit power p_k and channel attenuation h_k. For the sake of analysis, we suppose perfect estimation of both values.

At the output of the despreader, we have

$$y_k[m]|\,\tilde{\Delta}_k = \sum_{l=1}^{K} \frac{h_l e^{j\theta_l}\sqrt{2p_l}}{h_k\sqrt{2p_k}} c^{(l)}_{m+\Delta_l} \cdot c^{(k)}_{m+\tilde{\Delta}_k} + \nu[m] \cdot \frac{c^{(k)}_{m+\tilde{\Delta}_k}}{h_k\sqrt{2p_k}} \tag{14}$$

and, after accumulation over a code length M, we get

$$z_k[n]|\,\tilde{\Delta}_k = \sum_{m=0}^{M-1} y_k[m] = \mu_k \cdot e^{j\theta_k} + \zeta^{(\mathrm{MAI})}_k[n] + \zeta^{(\mathrm{AWGN})}_k[n], \tag{15}$$

where

$$\mu_k = \begin{cases} 1, & \tilde{\Delta}_k = \Delta_k, \\ 0, & \tilde{\Delta}_k \neq \Delta_k. \end{cases} \tag{16}$$

and where $\zeta^{(\mathrm{MAI})}_k[n]$ is the term arising from multiple access interference (MAI) inherent to (asynchronous) CDMA. The last term is due to AWGN and is clearly given by

$$\zeta^{(\mathrm{AWGN})}_k[n] = \zeta^{(\mathrm{AWGN})}_{I,k}[n] + j\zeta^{(\mathrm{AWGN})}_{Q,k}[n], \tag{17}$$

where

$$\zeta^{(\mathrm{AWGN})}_{I,k}[n], \zeta^{(\mathrm{AWGN})}_{Q,k}[n] \sim \mathcal{N}\left(0, \frac{N_0/T_c}{h_k^2 \cdot 2p_k \cdot M}\right). \tag{18}$$

By virtue of the central-limit theorem, the term due to the MAI can be approximated to a Gaussian random variable:

$$\zeta^{(\mathrm{MAI})}_k[n] = \zeta^{(\mathrm{MAI})}_{I,k}[n] + j\zeta^{(\mathrm{MAI})}_{Q,k}[n], \tag{19}$$

with

$$\zeta^{(\mathrm{MAI})}_{I,k}[n], \zeta^{(\mathrm{MAI})}_{Q,k}[n] \sim \mathcal{N}\left(0, \frac{\sum_{l\neq k} h_l^2 \cdot 2p_l/2}{h_k^2 \cdot 2p_k \cdot M}\right). \tag{20}$$

We can thus statistically characterize $z_k[n]|\,\tilde{\Delta}_k$ in a more compact form as follows:

$$\begin{aligned} z_k[n]|\,\tilde{\Delta}_k &= z_{I,k}[n]|\,\tilde{\Delta}_k + j\, z_{Q,k}[n]|\,\tilde{\Delta}_k, \\ z_{I,k}[n]|\,\tilde{\Delta}_k &\sim \mathcal{N}\left(\mu_k \cdot \cos\theta_k, \frac{1}{2\gamma_k}\right), \\ z_{Q,k}[n]|\,\tilde{\Delta}_k &\sim \mathcal{N}\left(\mu_k \cdot \sin\theta_k, \frac{1}{2\gamma_k}\right), \end{aligned} \tag{21}$$

where

$$\gamma_k = \frac{E^{(r)}_k}{I_{0,k} + N_0} = \frac{M \cdot h_k^2 p_k}{\sum_{l\neq k} h_l^2 p_l + \sigma^2} \tag{22}$$

is the SINR of user k, defined as the ratio between the energy per bit of user k collected at its receiver $E^{(r)}_k$ and the received PSDs due to MAI $I_{0,k}$ and to AWGN N_0, respectively.

We are now to detail the *synchronization strategy* shown in Fig. 6. Broadly speaking, this block combines $z_{I,k}[n]|\,\tilde{\Delta}_k$

and $z_{Q,k}[n]|\tilde{\Delta}_k$ to derive a final real-valued statistics $w_k[n]|\tilde{\Delta}_k;\rho_k$, to test acquisition. This quantity is a function of both the tentative code shift $\tilde{\Delta}_k$ (through the mean value of $z_k[n]$), and of the synchronization strategy that we choose, denoted by ρ_k, since the way the components of $z_k[n]$ are combined impacts on its probability density function (pdf). More details on practical synchronization strategies (e.g., coherent or non-coherent) will be provided in Section V.

To decide whether the k-th receiver is in-sync or out-of-sync, the output $w_k[n]|\tilde{\Delta}_k;\rho_k$ is compared with a *detection threshold* $\lambda_k[n] \in [0,1]$. For convenience of notation, we will drop the dependence of quantities such as decision statistics and threshold on the symbol index n from now on. In case the test fails, i.e., if $w_k|\tilde{\Delta}_k;\rho_k < \lambda_k$, then a new tentative code shift $\tilde{\Delta}_k$ is selected for the PN sequence generation (14). If the synchronization test is passed, i.e., if $w_k|\tilde{\Delta}_k;\rho_k > \lambda_k$, then the receiver assumes that the tentative delay $\tilde{\Delta}_k$ of the the locally generated PN sequence is the correct delay Δ_k and proceeds to verification mode [46], [47]. This process aims at avoiding false code locks, which are extremely detrimental for the receiver in terms of increased time for correct synchronization and subsequent data detection. However robust, verification is expensive in terms of time and processing resources. Hence, a key performance indicator of the acquisition strategy is given by the *probability of false alarm*, defined as

$$P_{FA}(\gamma_k,\lambda_k;\rho_k) = \Pr\left\{ w_k > \lambda_k | \tilde{\Delta}_k \neq \Delta_k;\rho_k \right\} = \int_{\lambda_k}^{+\infty} f_{W|\tilde{\Delta}_k\neq\Delta_k;\rho_k}(w)\,dw, \quad (23)$$

to be kept as low as possible. The twin performance parameter that fully characterizes the sync procedure is the *probability of detection*, defined as

$$P_{D}(\gamma_k,\lambda_k;\rho_k) = \Pr\left\{ w_k > \lambda_k | \tilde{\Delta}_k = \Delta_k;\rho_k \right\} = \int_{\lambda_k}^{+\infty} f_{W|\tilde{\Delta}_k=\Delta_k;\rho_k}(w)\,dw, \quad (24)$$

which on the contrary should be as high as possible, since a missed detection implies a much longer acquisition time and thus a significant delay in the receiving chain. In our definitions above, we expressed P_{FA} and P_D as explicit functions not only of the tentative code shift and of the adopted strategy, but also of the SINR γ_k through the variance of z_k.

From (23)-(24) we can extract general requirements for synchronization strategies to be amenable to our game-theoretic analysis. The first requirement is in terms of the behavior of P_{FA} and P_D versus the threshold $\lambda_k \in [0,1]$ for a fixed SINR γ_k, for all $k \in \{1,\cdots,K\}$: both P_{FA} and P_D must decrease as λ_k increases. This is a reasonable assumption, since, for a fixed SINR, increasing λ_k means increasing the lower interval of the integrals (23)-(24), thus reducing the interval of integration. The second requirement is in terms of the behavior of P_{FA} versus the SINR γ_k for a fixed threshold λ_k, for all $k \in \{1,\cdots,K\}$: P_{FA} must decrease as γ_k increases, with $\lim_{\gamma_k\to+\infty} P_{FA} = 0$. The third requirement is in terms of the behavior of P_D versus the SINR γ_k for a fixed threshold λ_k, for all $k \in \{1,\cdots,K\}$: P_D must increase as γ_k increases, with $\lim_{\gamma_k\to+\infty} P_D = 1$. Again, the last two requirements appear to be reasonable, since increasing γ_k tightens the pdf around its mean value, which is 0 in case of wrong code shift and 1 in case of correct code shift.

Under these assumptions (that are invariably satisfied by practical synchronization strategies), the performance of the considered system model, measured in terms of probabilities of detection and false alarm, increases as the SINR increases. As a consequence, there exists a tradeoff between good synchronization performance on one side, and low energy consumption on the other. This is analogous to numerous approaches in the field of resource allocation for data detection, as those described in Section II-C, in which the performance index is given by the effective throughput at the receiver.

IV. Formulation of the code-sync game

The tradeoff between good synchronization performance and energy saving, is captured if we define a utility function as the ratio between the probability of detection P_D and the transmitted energy per bit (or *per acquisition*): $E_k^{(t)} = p_k \cdot T_b$ as follows:

$$u_k(\mathbf{p},\lambda_k) = u_k\left((p_k,\mathbf{p}_{\setminus k}),\lambda_k\right) = \frac{P_D(\gamma_k,\lambda_k;\rho_k)}{p_k \cdot T_b}, \quad (25)$$

where $\mathbf{p}_{\setminus k} = \mathbf{p} \setminus p_k = (p_1,\cdots,p_{k-1},p_{k+1},\cdots,p_K)$. Each k-th transmitter-receiver pair can set its own transmit power p_k (at the transmitter side) and its detection threshold λ_k (at the receiver side). On the contrary, the synchronization strategy ρ_k is assumed to be given, since it depends on the availability of some a-priori information on the channel experienced by transmitter k, such as the residual phase offset θ_k, which may or may not be available at the k-th receiver.

Although $E_k^{(t)}$ is a function of p_k only, P_D depends on the achieved SINR at the receiver in addition to the threshold. Using (22), we can easily verify that γ_k depends not only on the value of p_k, but also on all other users' transmit powers $\mathbf{p}_{\setminus k}$. As a consequence, maximizing $u_k(\mathbf{p},\lambda_k)$ is not a unilateral optimization, but a multidimensional problem. In this context, we can formulate a noncooperative game with complete information [1] in which every transmitter-receiver pair seeks to maximize its own utility by choosing an optimum configuration in terms of (*transmit power*, *threshold*).

In addition to this, we have also to place a constraint on the (maximum) probability of false alarm $\overline{P}_{FA,k}$ to limit the occurrence rate of spurious detections. In some sense, $\overline{P}_{FA,k}$ represents user k's quality-of-service (QoS) requirement, that depends on the desired accuracy to be achieved. Let $\mathcal{G} = [\mathcal{K},\{\mathcal{A}_k\},\{u_k\}]$ be such game, in which $\mathcal{K} = \{1,\ldots,K\}$ is the index set for the transmitter-receiver pairs; $\mathcal{A}_k = \mathcal{P}_k \times \Lambda_k$ is the strategy set for user $k \in \mathcal{K}$, where $\mathcal{P}_k$ is the transmit power set, and Λ_k is the set of threshold values; and u_k is the payoff function for the pair $k \in \mathcal{K}$. The power strategy set is $\mathcal{P}_k = \left[\underline{p}_k,\overline{p}_k\right]$, with $\underline{p}_k$ and $\overline{p}_k$ denoting the minimum and maximum power constraints, respectively. The threshold strategy set is $\Lambda_k = [0,1]$ for all receivers $k \in \mathcal{K}$. Formally,

$\mathcal{G}$ can be expressed as

$$(p_k^*, \lambda_k^*) = \underset{p_k \in \mathcal{P}_k, \lambda_k \in \Lambda_k}{\arg\max} \; u_k(\mathbf{p}, \lambda_k) \quad \text{subject to} \quad P_{FA}(\gamma_k, \lambda_k; \rho_k) \le \overline{P}_{FA,k} \tag{26}$$

where $u_k(\mathbf{p}, \lambda_k)$ is defined as in (25). The optimality criterion (26) can be reformulated more conveniently by inverting (22):

$$p_k = \frac{\sum_{l \neq k} h_l^2 p_l + \sigma^2}{M \cdot h_k^2} \cdot \gamma_k = \frac{\gamma_k}{\xi_k}, \tag{27}$$

where ξ_k is independent of p_k and λ_k. Hence, (26) equals

$$(\gamma_k^*, \lambda_k^*) = \underset{\gamma_k \in [0, \xi_k \overline{p}_k], \lambda_k \in [0,1]}{\arg\max} \; \frac{P_D(\gamma_k, \lambda_k; \rho_k)}{\gamma_k}, \quad \text{subject to} \quad P_{FA}(\gamma_k, \lambda_k; \rho_k) \le \overline{P}_{FA,k} \tag{28}$$

with $\gamma_k^* = \xi_k p_k^*$.

Now, adding the constraint $P_{FA}(\gamma_k, \lambda_k; \rho_k) \le \overline{P}_{FA,k}$ actually means identifying that subset $\mathcal{A}'_k \subset [0, \xi_k \overline{p}_k] \times [0,1]$ that provides $P_{FA}(\gamma_k, \lambda_k; \rho_k) \le \overline{P}_{FA,k}$. Since P_{FA} is assumed to decrease as both λ_k and γ_k increase, we can set $\lambda_k = 1$ to identify the minimum SINR $\underline{\gamma}_k$ that provides $P_{FA}(\underline{\gamma}_k, \lambda_k = 1; \rho_k) = \overline{P}_{FA,k}$. For the sake of notation, we define a function $\underline{\gamma}(\cdot)$, depending on both the QoS constraint and the synchronization strategy, such that

$$\underline{\gamma}_k = \underline{\gamma}\left(\overline{P}_{FA,k}; \rho_k\right). \tag{29}$$

It is easy to verify that, for any $\lambda_k \in [0,1]$, a necessary condition for the QoS constraint to be verified is $\gamma_k \in \left[\underline{\gamma}_k, \xi_k \overline{p}_k\right]$. Given a fixed $\gamma_k \in \left[\underline{\gamma}_k, \xi_k \overline{p}_k\right]$, a subset $[\underline{\lambda}_k(\gamma_k), 1] \in \Lambda_k$ that fulfills $P_{FA}(\gamma_k, \lambda_k; \rho_k) \le \overline{P}_{FA,k}$ can also be obtained using similar arguments. Since P_{FA} is a decreasing function of λ_k, $\underline{\lambda}_k(\gamma_k)$ is such that $P_{FA}(\gamma_k, \lambda_k = \underline{\lambda}_k(\gamma_k); \rho_k) = \overline{P}_{FA,k}$. Similarly, we define a function $\underline{\lambda}(\cdot)$ such that

$$\underline{\lambda}_k(\gamma_k) = \underline{\lambda}\left(\gamma_k, \underline{\gamma}_k; \rho_k\right) \tag{30}$$

that accounts for the QoS constraint and the receiver type. Based on the customary behavior of P_{FA}, $\underline{\lambda}_k(\gamma_k)$ increases as $\underline{\gamma}_k$ increases, while it decreases as γ_k increases. The functions $\underline{\gamma}(\cdot)$ and $\underline{\lambda}(\cdot)$ will be explicitly characterized in Section V. Note that, due to the decreasing monotonic behavior of P_{FA} with respect to both λ_k and γ_k, both $\underline{\gamma}(\cdot)$ and $\underline{\lambda}(\cdot)$ are bijective functions.

Finally, our game (28) can be cast into the following compact form:

$$(\gamma_k^*, \lambda_k^*) = \underset{\gamma_k \in \left[\underline{\gamma}_k, \xi_k \overline{p}_k\right], \lambda_k \in \left[\underline{\lambda}_k(\gamma_k), 1\right]}{\arg\max} \; \frac{P_D(\gamma_k, \lambda_k; \rho_k)}{\gamma_k}. \tag{31}$$

From now on, we will replace $P_D(\gamma_k, \underline{\lambda}_k(\gamma_k); \rho_k)$ with $P_D(\gamma_k; \underline{\gamma}_k, \rho_k)$ and we explicitly notice that our formulation includes: i) the energy-efficient tradeoff in terms of good synchronization performance versus energy consumption; ii) the synchronization strategy chosen; and iii) the QoS constraint in terms of probability of false alarm.

It is time now to proceed to solve our resource allocation problem (31) using the analytical tools of game theory. In particular, we search for the Nash equilibria of the game $\mathcal{G}$, i.e, those pairs (p_k^*, λ_k^*) such that, for all $k \in \mathcal{K}$,

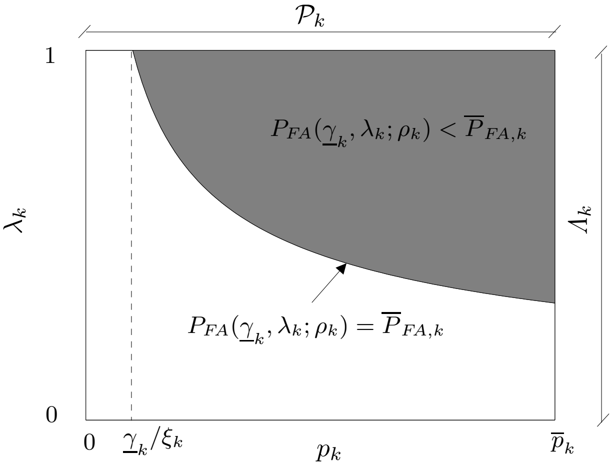

Fig. 7. A typical strategy set $\mathcal{A}'_k$ over the 2-D resource plan $\mathcal{P}_k \times \Lambda_k$.

$$u_k\left(\left(p_k^*, \mathbf{p}_{\setminus k}^*\right), \lambda_k^*\right) \ge u_k\left(\left(p_k, \mathbf{p}_{\setminus k}^*\right), \lambda_k\right) \tag{32}$$

for all transmit powers $p_k \in \mathcal{P}_k$ *and* for all thresholds $\lambda_k \in \Lambda_k$, and such that $P_{FA}(\gamma_k, \lambda_k; \rho_k) \le \overline{P}_{FA,k}$. We will formulate our main results in terms of theorems.

Theorem 1: The game $\mathcal{G}$ admits (at least) one NE if the following two conditions are met:

$$P_D''\left(\gamma_k; \underline{\gamma}_k, \rho_k\right) = \frac{d^2}{d\gamma_k^2} P_D(\gamma_k; \underline{\gamma}_k, \rho_k) < 0 \tag{33a}$$

$$\Phi = \sum_{k=1}^{K} \varphi_k < 1 \tag{33b}$$

where

$$\varphi_k = \left(\frac{M}{\gamma_k^*} + 1\right)^{-1} > 0, \tag{34}$$

and where

$$\gamma_k^* = \begin{cases} \underline{\gamma}_k, & \underline{\gamma}_k \le h\left(\underline{\gamma}_k; \underline{\gamma}_k, \rho_k\right), \\ \tilde{\gamma}_k, & \underline{\gamma}_k > h\left(\underline{\gamma}_k; \underline{\gamma}_k, \rho_k\right), \end{cases} \tag{35}$$

is the SINR that maximizes $u_k(\mathbf{p}, \lambda_k)$, with $\tilde{\gamma}_k > \underline{\gamma}_k$ satisfying the relation $\tilde{\gamma}_k = h\left(\tilde{\gamma}_k; \underline{\gamma}_k, \rho_k\right)$, and $h\left(\gamma_k; \underline{\gamma}_k, \rho_k\right) \triangleq P_D(\gamma_k; \underline{\gamma}_k, \rho_k) \Big/ [dP_D(\gamma_k; \underline{\gamma}_k, \rho_k)/d\gamma_k]$.

The proof, omitted for the sake of brevity, is made in two steps, showing that i) $u_k(\mathbf{p}, \lambda_k)$ is continuous and quasi-concave in $(p_k, \lambda_k) \in \mathcal{A}'_k = \left[\underline{\gamma}_k, \xi_k \overline{p}_k\right] \times \left[\underline{\lambda}_k(\gamma_k), 1\right] \subset \mathcal{A}_k$; and ii) $\mathcal{A}'_k$ is a nonempty, convex, and compact subset of some Euclidean space. A typical $\mathcal{A}'_k$ is given by the shadowed area, including its contour, of Fig. 7. This condition is sufficient to ensure the quasi-concavity of $u_k(\mathbf{p}, \lambda_k)$.

As can be easily verified, (33a) represents a rather loose constraint to the shape of P_D as a function of ρ_k. On the contrary, the requirement (33b) is a necessary and sufficient condition for $\mathcal{A}'_k$ to be nonempty, and implies some form of *admission control*, to be performed before the game starts to allow all players to achieve their target SINRs γ_k^*. In the case that $\Phi \geq 1$, there will be a subset $\mathcal{K}'$ of players that will achieve γ_k^*, whilst the remaining $\mathcal{K} \setminus \mathcal{K}'$ will not. The quantity φ_k can be thought of as the *size* that player k (i.e., the k-th transmitter-receiver pair) occupies in the space of the available resources (the bidimensional space $\mathcal{A}_k = \mathcal{P}_k \times \Lambda_k$). We can show that γ_k^* (and thus p_k^*) increases as φ_k increases. Hence, the larger φ_k, the larger amount of such resources "consumed" [9], [48].

To derive other properties of the NE, including its uniqueness, we consider it from another point of view, focusing on the transmit power p_k. The power level chosen by a *rational* self-optimizing player is its *best response* to the powers $\mathbf{p}_{\setminus k}$ chosen by the other players (any threshold, including receiver k's λ_k, does not affect this choice, due to the monotonic decreasing behavior of P_D with λ_k). Formally, player k's best response is the map that assigns each $\mathbf{p}_{\setminus k} \in \mathcal{P}_{\setminus k}$ the set

$$r_k\left(\mathbf{p}_{\setminus k}\right) = \left\{p_k \in \mathcal{P}_k : u_k\left(\left(p_k, \mathbf{p}_{\setminus k}\right), \lambda_k\right) \geq u_k\left(\left(p_k', \mathbf{p}_{\setminus k}\right), \lambda_k\right) \quad \text{for all } p_k' \in \mathcal{P}_k\right\}, \quad (36)$$

where $\mathcal{P}_{\setminus k}$ is the strategy space of all users excluding user k. With the notion of a player's best response, the transmit power at the NE can be restated in a compact form: $\mathbf{p}^*$ is the vector of transmit powers at the NE of the game $\mathcal{G}$ for all $k \in \mathcal{K}$.

Theorem 2: The game $\mathcal{G}$ has a unique NE, achieved when

$$\lambda_k^* = \underline{\lambda}_k\left(\gamma_k^*\right) \quad (37)$$

$$p_k^* = \frac{\gamma_k^*}{\xi_k} = \frac{\varphi_k}{h_k^2} \cdot \frac{\sigma^2}{1-\Phi}, \quad (38)$$

where γ_k^* is defined as in (35). The proof, omitted for brevity again, can be obtained showing that the correspondence $\mathbf{r}\left(\mathbf{p}^*\right) = \left(r_1\left(\mathbf{p}_{\setminus 1}^*\right), \cdots, r_K\left(\mathbf{p}_{\setminus K}^*\right)\right)$ is a *standard* function [42].

It is interesting to note that the expression (38) for the level of transmit power at the NE is also applicable to the case of the data-detection-oriented power control scheme illustrated in Section II-C. Using the analysis reported in [7], we can show that p_k^* is a function of φ_k and Φ, provided that the definition (34) makes use of the target SINR γ_k^* in accordance with the utility function (5). So the *criterion* to perform optimal resource allocation for code synchronization is the same as the one for data detection, but the target SINRs and the resulting power levels may turn out to be different in the two cases, according to the different requirements coming either from data detection or from synchronization. The difference in terms of power levels at the NE between the two cases derives form the fact that we must replace γ_k^* that follows from (35) with the SINR that maximizes (5). In the practice, during the initial phase of code synchronization we use γ_k^* in accordance with (35), and we *jointly* set λ_k^* and p_k^* following (37)-(38) to maximize the probability of correct code acquisition per transmitted energy consumed, without caring of the achieved throughput at the receiver. After acquisition is over, we will use the target SINR that maximizes (5) to maximize the goodput at the receiver per energy consumed at the transmitter, neglecting the (now irrelevant) performance of the receiver during acquisition.

TABLE I
MATHEMATICAL DESCRIPTION OF PRACTICAL SYNCHRONIZERS.

type	coherent	non-coherent (mod-ℓ)
identifier	$\rho_k = 0$	$\rho_k = \ell$
strategy	$w_k = z_k e^{j\theta_k}$	$w_k = \lvert z_k \rvert^\ell$
$P_{FA}(\gamma_k, \lambda_k; \rho_k)$	$Q\left(\lambda_k \sqrt{2\gamma_k}\right)$	$e^{-\lambda_k^{2/\ell}\gamma_k}$
$P_D(\gamma_k, \lambda_k; \rho_k)$	$1 - Q\left((1-\lambda_k)\sqrt{2\gamma_k}\right)$	$Q_1\left(\sqrt{2\gamma_k}, \lambda_k^{1/\ell}\sqrt{2\gamma_k}\right)$
$\underline{\gamma}\left(\overline{P}_{FA,k}; \rho_k\right)$	$\frac{1}{2}\left[Q^{-1}\left(\overline{P}_{FA,k}\right)\right]^2$	$-\log \overline{P}_{FA,k}$
$\underline{\lambda}\left(\gamma_k, \underline{\gamma}_k; \rho_k\right)$	$\sqrt{\underline{\gamma}_k/\gamma_k}$	$\left(\underline{\gamma}_k/\gamma_k\right)^{\ell/2}$

Similarly to the case of the energy-efficient power control described in Section II-C, the NE for code synchronization is not Pareto-optimal. We could use pricing techniques similar to that employed for data-detection-based schemes, to obtain Nash solutions which are superior in the sense of Pareto-optimality.

V. SIMULATION RESULTS

We provide in this section some numerical results to support and integrate the theoretical analysis presented in Section IV. Our iterative distributed algorithm (not reported in detail for brevity) is a straightforward implementation of the best response outlined above, and it is similar to the update criterion (8) reported in Section II-C for the case of data detection. Note that, using the theoretical formulation of [42], we can easily show that the decentralized best-response algorithm always converge to the NE. The only information that must be provided by the access point, as it is not locally available at the transmitter, is the current received SINR level, which can be fed back by the receiver with a very modest data rate requirement on the signaling channel. An issue still open in the current formulation is the way the receiver measures the SINR level when the code synchronization is not yet achieved. The following simulations are thus conducted assuming a genie-aided framework that allows each transmitter to know its received SINR at the concentration point. The same considerations applies to the case of coherent detectors, since it is very unlikely that a good carrier phase estimate (required by coherent detection) can be obtained prior to achieving code lock and performing signal despreading.

Table I specifies the details of our analysis both for an ideal *coherent* synchronizer, in which θ_k is perfectly known at the receiver, and for a non-coherent synchronizer, in which θ_k is unknown (in the proposed simulations, we use $\ell = 1$ and $\ell = 2$). The functions $Q(x) = \frac{1}{2\pi}\int_x^{+\infty} \exp\left(-t^2/2\right) dt$ and $Q_1(\alpha, x) = \int_x^{+\infty} t \cdot \exp\left[-\left(t^2+\alpha^2\right)/2\right] I_0(\alpha t)\, dt$ are the complementary cumulative distribution function (cdf) of a standard random variable and the Marcum's Q-function [49], respectively, with $I_0(\cdot)$ denoting the modified Bessel function of the first kind and order 0. Note that (33a) is fulfilled by both classes.

Throughout our simulations, the noise power is assumed to be $\sigma^2 = 50\,\text{nW}$, whereas the maximum power constraint is $\overline{p}_k = \overline{p} = 0.5\,\text{W}$. The distance d_k between the k-th transmitter and the access point is uniformly distributed between 3 and

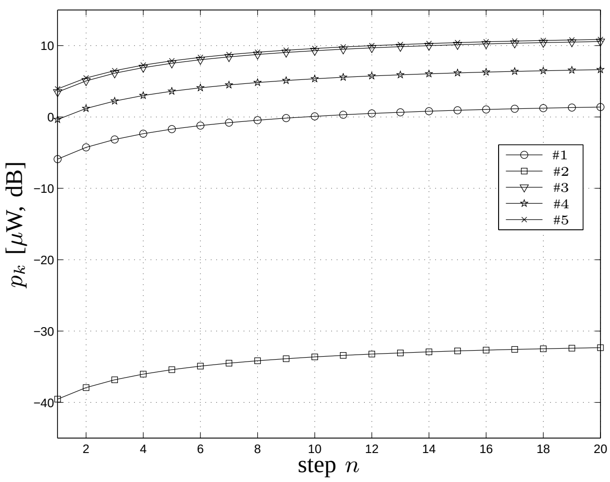

Fig. 8. Transmit power as a function of the iteration step.

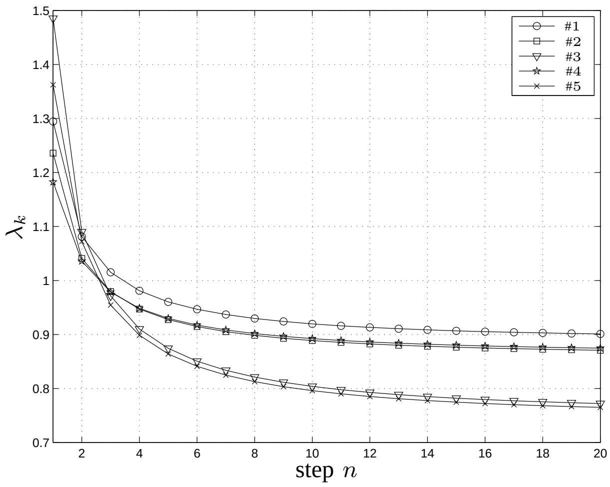

Fig. 9. Detection threshold as a function of the iteration step.

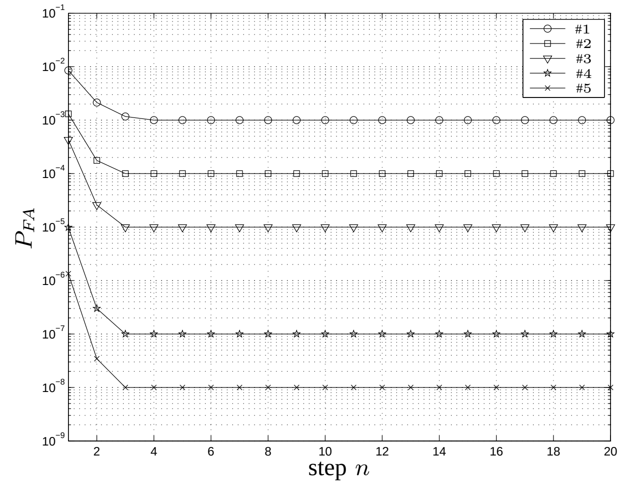

Fig. 10. Probability of false alarm as a function of the iteration step.

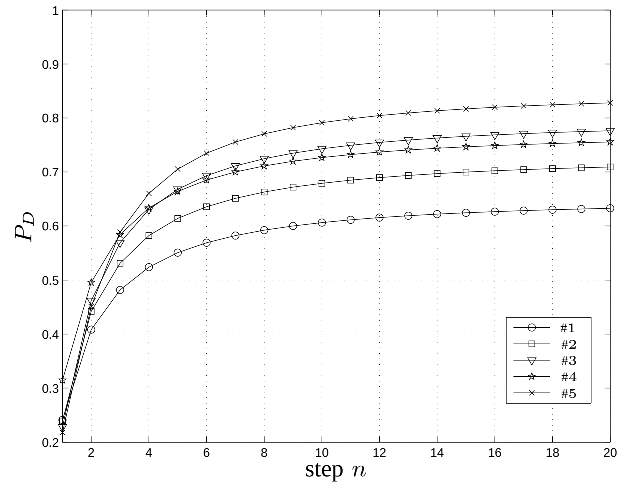

Fig. 11. Probability of detection as a function of the iteration step.

$100\,\mathrm{m}$. The channel gains h_k are assumed to be Rayleigh-distributed, with $\mathbb{E}\left\{h_k^2\right\} = 1.5 \cdot (d_0/d_k)^2$, where $d_0 = 10\,\mathrm{m}$ is the reference distance between the transmitter and the receiver. The Rayleigh distribution is adopted to emulate the effects of shadowing to some extent.

Figs. 8 and 9 show the behavior of the joint threshold and power control as a function of the iteration step n. The results have been obtained using a random realization of the network with spreading factor $M = 64$ and $K = 5$ users. Note that these parameters have been intentionally kept low for the sake of graphical presentation. The users make use of the synchronization strategies $\rho_k = \{0, 0, 2, 1, 2\}$, and place the QoS requirements $\overline{P}_{FA,k} = \left\{10^{-3}, 10^{-4}, 10^{-5}, 10^{-7}, 10^{-8}\right\}$. Given this configuration, $\underline{\gamma}_k = \{6.8, 8.4, 10.6, 12.1, 12.7\}$ dB, and $\gamma_k^* = \{7.8, 9.7, 11.8, 13.3, 13.9\}$ dB. Note that condition (33b) holds, since $\Phi \cong 0.93 < 1$. In this snapshot, $h_k = \{0.19, 11.55, 0.10, 0.18, 0.12\}$. According to (37) and (38), $\lambda_k^* = \{0.89, 0.86, 0.76, 0.87, 0.75\}$ and $p_k^* = \{1.6, 6.8\text{e-}4, 13.3, 5.4, 14.2\}$ μW. As can be seen, the terminals rapidly converge to the desired pairs (p_k^*, λ_k^*), with a convergence rate of the algorithm proportional to $1/(1-\Phi)$. At the beginning of the procedure, $\lambda_k > 1$ for some users. This means that the QoS constraint is not met. As a consequence, such terminals cannot apply the desired synchronization strategy (their detectors are switched off), and they remain in an initial phase of power control only. However, after a few steps, $\lambda_k \leq 1$, which yields $P_{FA,k} \leq \overline{P}_{FA,k}$. This can be easily verified in Fig. 10, which reports the behavior of P_{FA} as a function of the iteration step. In this network configuration, after the fourth step all users meet their QoS requirements, while maximizing P_D (Fig. 11).

VI. Summary and Conclusion

After a broad introduction to the topic of game-theoretic resource allocation in wireless communication, this paper investigated the issue of game-theory-based criteria to optimize the function of initial code acquisition in a CDMA network. Using the tools of game theory, the problem was restated as a noncooperative (distributed) game in which the transmitter-receiver pairs (the players) jointly set their transmit powers (at the transmitter side) and detection thresholds (at the receiver side) to maximize the ratio between the probability of detection and the transmitted energy per acquisition. Users are also supposed to place QoS requirements in terms of maximum probability of false alarm and to choose their preferred synchronization strategy (coherent, noncoherent) based on the possible knowledge of some a-priori parameters. General properties of the Nash equilibrium of the game were investigated, also deriving explicit expressions for the strategies at the equilibrium as functions of the network configuration, and sketching a possible implementation for a distributed algorithm.

The main conclusion is that the Nash solution for energy-efficient distributed synchronization based on maximum detection probability per consumed Joule of energy can be found following a criterion that is similar to the one used for energy-efficient data detection based on maximum throughput per Joule. Nonetheless, the two cases show significant differences when applied to the same CDMA scenario, leading to different values of optimum transmitted power by the mobile terminals. The two solutions may be not in conflict in a network set-up scenario, since the criterion for optimum synchronization will be used in a first instance, to revert to the data-detection optimal criterion after synchronization is over. Further research is needed to assess the case of a mixed population, i.e., some terminals already in-sync and using the data-optimum criterion co-existing with other terminals still in the acquisition phase.

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