Are Carrier Transport Effects Important for Chirp Modeling of Quantum-Well Lasers?

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***Abstract*—The paper investigates the impact of carrier transport effects on the chirp modeling of quantum-well lasers. Particularly, the difference between the full modeling based on quantum-well laser rate equations is compared with modeling based on formulas derived for bulk lasers. As it was shown, the relations between chirp and intensity modulation are quite similar in both cases.**

***Index Terms*—laser chirp, laser modeling**

I. INTRODUCTION

THE quantum-, or multi-quantum-well (QW, or MQW) structure introduced to the semiconductor laser design implies some new phenomena in the device operation, when compared with the bulk laser design. Among them the transport of injected carriers across the separate-confinement-heterostructure (SCH) and capturing them into the QW regions introduce some delay in the carriers flow. Consequently, noticeable variations of the concentration of carriers accumulated in SCH region occur. Because a large fraction of the optical mode lies in the SCH, this carrier density variations affect the lasing frequency i.e. introduces a new chirp component.

There are plenty of papers in which significant differences in chirp characteristics of bulk and QW lasers are pointed out [1]–[4]. On the other hand, there are some papers in which the QW laser chirp is modeled using equations derived for bulk device. In some of them the considerations are verified by experiments, which seems to proof such chirp treatment [5]–[7]. The aim of the work presented herein is to clarify this confusing inconsistency and to point out the area in which the simple chirp model may be used for QW lasers.

II. THEORETICAL BASICS

The basic mathematical model of semiconductor laser is the set of rate equations, which describes the dynamics of carrier and photon densities, and relate them to the laser frequency chirp and the output optical power.

A. Bulk laser modeling

For the bulk laser the rate equations may be written in the following form:

$$\frac{dN}{dt} = \frac{I}{eV_a} - \frac{N}{\tau_e} - \frac{g_0(N - N_T)}{1 + \varepsilon_g S} S \tag{1}$$

$$\frac{dS}{dt} = \frac{\Gamma g_0(N - N_T)}{1 + \varepsilon_g S} S - \frac{S}{\tau_P} + \frac{\Gamma \beta N}{\tau_e} \tag{2}$$

$$\Delta\nu = \frac{\alpha}{4\pi} \Gamma g_0 (N - N_{TH}) \tag{3}$$

$$P = \frac{\eta V_a h \nu_0}{\Gamma \tau_p} S \tag{4}$$

where N is the carrier concentration in the active region, S is the photon concentration, I is the injected current, e is the electron charge, V_a is the active region volume, τ_e is the carrier lifetime, g_0 is the differential gain, ε_g is the gain compression factor, N_T is the carrier concentration for transparency, N_{TH} is threshold carrier concentration, Γ is the confinement factor, τ_p is the photon lifetime, β is the spontaneous emission coefficient, $\Delta\nu$ is the optical frequency deviation (i.e. the chirp), α is the line enhancement factor, P is the output power, h is Planc's constant, and ν_0 is the nominal optical frequency.

As may be noticed, the frequency chirp is described by (3), which shows that the frequency deviation is proportional to the concentration of carriers in the laser active region.

A serious practical drawback of the (3) is that it relates the chirp to the unobservable carrier concentration, which cannot be predicted without the precise knowledge about all the rate equations parameters. Thus, it is very useful to relate the chirp to the measurable laser output power. Calculating the carrier concentration N from (2) and putting it into (3), the frequency chirp may be related to the photon concentration. Ignoring some negligible terms and using (4), we may finally relate the chirp to the laser output power:

$$\Delta\nu(t) = \frac{\alpha}{4\pi} \left(\frac{1}{P(t)} \frac{dP(t)}{dt} + \kappa P(t) \right) \tag{5}$$

where $\kappa = \Gamma \varepsilon_g / (\eta V_a h \nu_0)$ is the so called adiabatic chirp coefficient. The part of the chirp induced by the time derivate of power is called the dynamic chirp, and the part directly proportional to the power is called the adiabatic one.

In case of small signal laser modulation, the frequency modulation (FM) efficiency may be determined using (5). In the frequency domain it takes the form:

$$\frac{\delta\nu(\omega_m)}{\delta I(\omega_m)} = \frac{\alpha}{4\pi} \left(\frac{j\omega_m}{\langle P \rangle} + \kappa \right) \frac{\delta P(\omega_m)}{\delta I(\omega_m)} \tag{6}$$

where $\delta(\cdot)$ denotes the small signal component of each quantity, ω_m is the angular frequency of laser modulation, $\langle P \rangle$ isthe mean optical power, and $\delta P(\omega_m)/\delta I(\omega_m)$ is the intensity modulation (IM) efficiency.

Thus, having the knowledge about the laser IM behavior (some kind of model or measured data) we need only two parameters (α and κ) to accurate chirp characterization. Some relatively simple measurement methods for determining these parameters are described in many papers [8].

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B. QW laser modeling

In the QW lasers the carrier concentrations in SCH and QW regions should be distinguished, and thus two separate rate equations for the carriers are introduced:

$$\frac{dN_b}{dt} = \frac{I}{eV_w} - \frac{N_b}{\tau_{cap}} - \frac{N_b}{\tau_e} + \frac{N_w}{\tau_{esc}} \tag{7}$$

$$\frac{dN_w}{dt} = \frac{N_b}{\tau_{cap}} - \frac{N_w}{\tau_{esc}} - \frac{N_w}{\tau_e} - \frac{g_0(N_w - N_T)}{1 + \varepsilon_g S}S \tag{8}$$

where N_w is the carrier concentration in the quantum wells, N_b is some equivalent concentration related with the real SCH carrier concentration N_s by the relation: $N_b = N_s V_s / V_w$, in which V_s and V_w are the volumes of SCH and QW, respectively. The capturing of the carriers from SCH to QW is characterized by capture time τ_{cap}, and (much less efficient) escaping in the opposite direction by τ_{esc}. The photon density depends only on the N_w concentration, thus:

$$\frac{dS}{dt} = \frac{\Gamma g_0(N_w - N_T)}{1 + \varepsilon_g S}S - \frac{S}{\tau_P} + \frac{\Gamma \beta N_w}{\tau_e} \tag{9}$$

The frequency chirp depends on both QW and SCH carrier densities, because the optical field lies in both regions undergoing carrier concentration variations. Thus, the chirp may be expressed as follows [1]:

$$\Delta\nu = \frac{\alpha}{4\pi}\Gamma g_0(N_w - N_{wTH}) + (1-\Gamma) g_b (N_b - N_{bTH}) \tag{10}$$

where N_{wTH} and N_{bTH} are threshold carrier concentrations in QW and SCH, respectively, g_b is the coefficient characterizing the efficiency of influence of N_b on the laser frequency.

Unfortunately, this time the chirp cannot be easily related to the intensity modulation, as it was made in (5) and (6) for the bulk lasers. Large signal relation, analogous to (5), is quite complicated, and even after many simplifications needs at least four parameter values to be determined in some way. Similarly, the small signal relation analogous to (6) is also troublesome and needs a large set of parameters [1].

Thus, the question of practical importance arises whether a relatively simple model of the laser IM and FM properties, based on the bulk laser rate equations, may be adopted for behavioral (i.e. not strictly connected with physical phenomena) modeling of the QW lasers.

In case of IM characteristics, is was shown in [7] that the effects arising from the carrier accumulation in the SCH may be simply modeled by a first order low-pass filter with time constant equal to τ_{cap}, preceding the bulk model of the inner QW structure. It may be also shown that for QW lasers with any low capture time the difference in the IM properties of models described by Eqs. (1), (2) and (7) ... (9) practically vanishes.

III. SMALL-SIGNAL CONSIDERATIONS

First, the small-signal chirp characteristics arising from the QW laser model based on the rate equations (7) ... (10) will be analyzed. Using this model and starting from two experimentally verified sets of its parameters, taken from [9], the laser FM efficiency versus modulation frequency was obtained. In some initial investigations it was observed that under the reasonable assumption that $\tau_{cap} << \tau_{esc}$, τ_{cap} becomes the crucial parameter determining both IM and FM differences between the QW and bulk laser model. This is illustrated in Fig. 1 and Fig. 2. For the capture time not greater than a few ps, the rate equation modeling the SCH region carrier concentration has no considerable influence on laser IM and FM properties and may be eliminated, which leads back to the bulk laser model. For higher values of τ_{cap} the IM modulation bandwidth reduces, which is in consistence with the above mentioned low-pass-filter-like behavior of SCH. A bit more complicated is the influence of the capture time on FM properties. It may be observed that for low modulation frequencies high values of τ_{cap} increase FM efficiency but for high frequencies the influence is opposite. It may be explained as follows: for low modulation frequencies greater values of τ_{cap} lead to higher modulation of carrier concentration in SCH region, which results in higher chirp. For higher frequencies, however, the low-pass filtering nature of SCH carrier accumulation reduces the fluctuations of carrier concentrations in SCH and, consequently, also in QW regions. This way, the chirp is strongly reduced (similarly to the IM response).

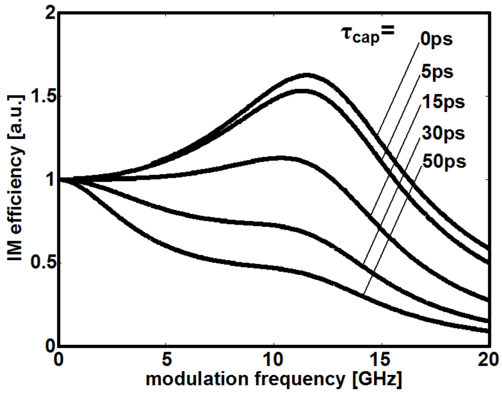

Fig. 1. . IM efficiency $|\delta P/\delta I|$ versus modulation frequency and capture time.

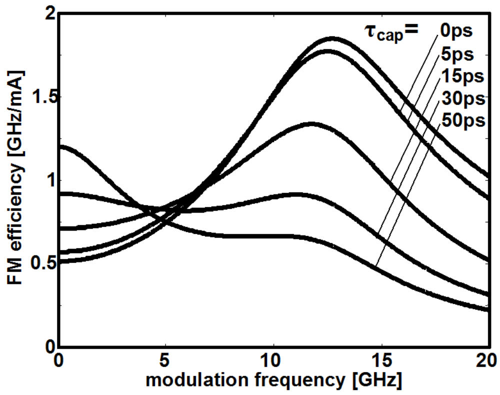

Fig. 2. FM efficiency $|\delta\nu/\delta I|$ versus modulation frequency and capture time.

Now, we come back to the practical question whether the (6) may be used as a simple behavioural model of QW laser chirp. In Fig. 3 the FM efficiency obtained from the full QW model is compared with that obtained using (6). Because (6) is now treated as a behavioural model, the adiabatic chirp coefficient

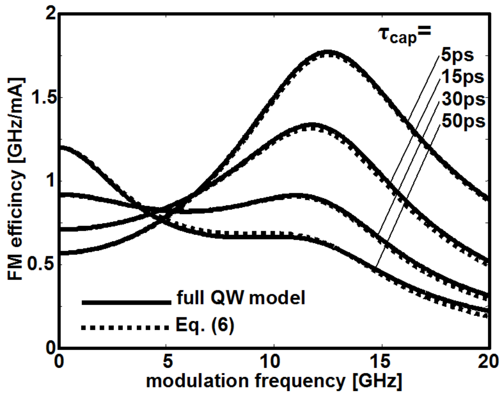

Fig. 3. Comparison of FM efficiency obtained from full QW model and from (6).

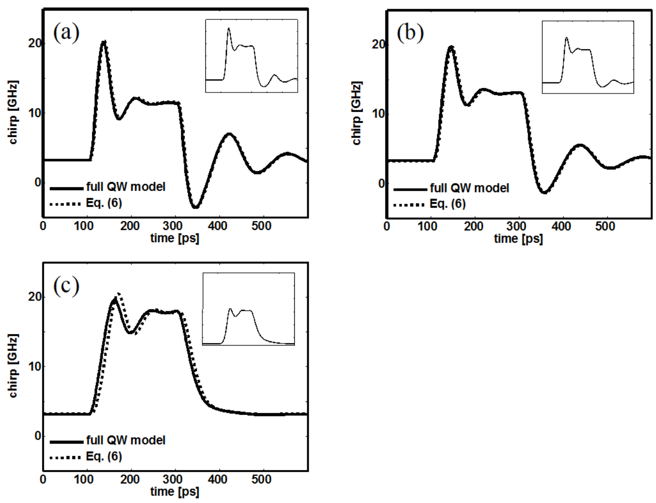

Fig. 4. Comparison of time domain chirp evolution obtained from full QW model and from (6); capture time equal to 5 ps (a), 15 ps (b) and 50 ps (c). In the insets corresponding power waveforms.

κ was trimmed to obtain a desired value of the low frequency chirp for each value of the taken capture time. It should be also pointed out that the IM response $\delta P(\omega_m)/\delta I(\omega_m)$ was modified each time by taking the actual one obtained from full QW rate equations model. As may be noticed, a very good agreement between the chirp obtained from the full model and from 6) was obtained, even for frequencies far above the laser relaxation frequency.

Concluding, the QW laser small-signal chirp may be accurately determined by the simple formula given in (6). However, the accurate IM response (known from any kind of model or measured data) is crucial for good accuracy.

IV. Large-Signal Considerations

The small-signal FM response is a basic laser property in any transmission system based on frequency/phase modulation, as some coherent or dispersion-supported systems. But also in case of systems based on direct intensity modulation, the laser chirp may be important when it interacts with the transmission channel chromatic dispersion. This time, however, rather large signal chirp properties should be analyzed.

Natural extension of the above presented small-signal considerations would be that also large-signal relation between bulk laser FM and IM may be adopted to QW lasers. Following the previous strategy, the large-signal laser chirp was determined by simulating the full QW rate equations model, and next compared with the chirp obtained from (5). As previously, the adiabatic chirp coefficient was trimmed to obtain the best agreement with the full model. The results are illustrated in Fig. 4 for various capture time values. The laser model was driven by the 200 ps long, nearly-rectangular current pulse. One may notice that the chirp obtained from (5) is extremely close to that resulting from the full model. Only for very large capture time, as 50 ps, some quite small delay (about 8 ps) may be observed in the chirp obtained from (5).

A very good agreement of the QW laser chirp characteristics obtained from the full model with that determined from (5) and (6) is somewhat surprising when we have in mind that they are derived from the bulk laser model. However, some intuitive explanation may be proposed. First, it should be noticed that using the "bulk" equations (5) and (6), the chirp induced in SCH region is "pushed" into the adiabatic chirp of the active region. This way the changes of the SCH carrier density (which in fact make the SCH chirp component) were in the model "substituted" by the changes of laser optical power, which in case of high-speed modulation would not exactly follow the SCH carrier density. Considering the case of large capture time first, we whould recall its low-pass filtering feature. The 50 ps capture time induces about 3 GHz cut-off, which depresses fast changes in the SCH carrier density. In this situation the "inner" laser is fast enough, and so the optical power nearly exactly follows the SCH carrier density, which explains the simple models accuracy.

For lower values of capture time the optical power may be more mismatched from SCH carrier density. But, on the other hand, small capture time results in small carrier accumulation in the SCH and so small chirp component caused by The SCH region. This way even less accurate modeling of this component has no significant influence on total chirp, and the simplified model is still quite accurate.

V. Experimental Verification

Direct measurement of large-signal time-resolved laser chirp is quite complicated and usually suffers from inherent bandwidth limitation introduced by the frequency response of FM/IM converting optical filters. Some indirect but quite precise verification of chirp modeling may be, however, performed based on the optical fiber chromatic dispersion. The interaction of the laser chirp with the fiber dispersion causes serious distortions in the time evolution of optical power detected at the fiber end. Comparing the distortions of the measured signal with that calculated based on the taken chirp model, its adequacy may be verified. The results of such experiment are shown in Fig. 5. The high-speed IM modulated signal (a piece of 10 Gb/s data stream) outgoing the MQW DFB laser (PT3563 type) is illustrated in Fig. 5(a). Taking the chirp model in the form of (5), with parameters α and κ obtained in other measurements, the chirp caused signal

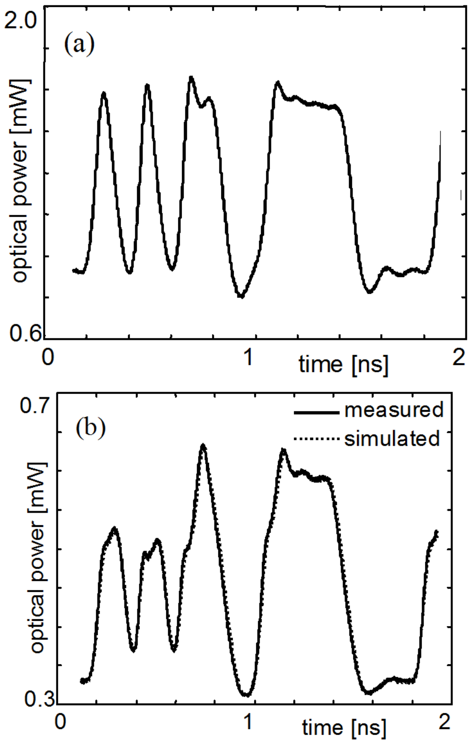

Fig. 5. Modulated laser output power (a), and fiber output power corrupted by interplay of the laser chirp and the fiber chromatic dispersion (b).

distortions after the 20 km long fiber were calculated, and compared with the measurement. As it is visible in Fig. 5(b), the calculated and measured fiber output signals are practically identical, which proves the adequate chirp modeling.

VI. CONCLUSIONS

The influence of the carrier transport between the SCH and QW regions is analyzed in the paper in the context of chirp modeling. It was shown that even for high values of carrier capture time, when the transport effects seriously affect the laser IM and FM characteristics, the simple relations coupling intensity modulation with chirp, derived for bulk lasers, may be used. It is of serious practical importance because it allowed us to determine the chirp from IM characteristics, using the model requiring only two parameters: the line enhancement factor and the adiabatic chirp coefficient. Namely, the time domain evolution of chirp may be obtained from the measured (or somehow modeled) time domain evolution of the laser output power, by means of (5). Alternatively, the frequency domain FM transfer function may be obtained from the frequency domain IM transfer function, using (6). This way in many cases the troublesome full QW laser modeling may be omitted without sacrificing the accuracy of considerations.

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