Methods of Real-time Calculation of Allan Deviation and Time Deviation

Andrzej Dobrogowski and Michał Kasznia

***Abstract*—The methods enabling real-time calculation of two commonly used parameters of timing signals – Allan deviation (ADEV) and time deviation (TDEV) – are presented in this paper. The idea of real-time computation of both parameters is described. The results of experimental tests of the methods enabling separate as well as joint real-time ADEV and TDEV computation are presented and discussed.**

***Index Terms*—timing signal, time error, Allan deviation, time deviation**

I. Introduction

The Allan deviation ADEV and time deviation TDEV allow the type of phase noise affecting the timing signal to be recognized. The parameters are commonly used for evaluation of signals generated by atomic clocks as well as for describing the quality of synchronization signal in the telecommunication networks [1]–[3]. The evaluation of the synchronization signal is commonly a two-stage process. First, the sequence of time error samples between the analyzed signal and some reference has to be measured at some network interface. When the measurement is completed, the calculation of the parameter's estimate using time error samples is performed. Such procedure causes an obvious delay in the evaluation process.

This paper describes the real-time methods of ADEV and TDEV computation, which enable the reduction of the evaluation time. These methods allow the estimates of ADEV and TDEV (which characterizes of more complex estimator's formula) to be computed in the real time, during the measurement process, simultaneously for a set of observation intervals. Additionally, the computation process can be performed jointly for both parameters.

In order to calculate the ADEV and TDEV estimate simultaneously for several observation intervals in the real time, all necessary operations should be performed in the time period between two sampling instants, i.e. during the sampling interval τ_0. The ability of performing the real-time assessment depends on several conditions: computation ability of the measurement equipment, sampling interval and number of the observation intervals considered. The methods described in the paper are developed for a measuring system where the time error counter and the computer controlling the measurement are two separate units. Therefore, the computer may be changed depending on the computing requirements. The results of experimental tests of the methods proposed for different conditions are presented in the paper. The calculations were performed for the time error sequences taken with sampling interval $\tau_0 = 1/30$ s, which is often used in the telecommunication applications. Different numbers and lengths of observation intervals simultaneously analyzed were considered.

II. Allan Deviation and Time Deviation

The computations of the Allan deviation and time deviation estimates are based on the averaging of second differences of the phase process $x(t)$ of the analyzed timing signal. We can assume for the telecommunication applications, in the case of negligible influence of frequency drift, that ADEV and TDEV are estimated based on the time error function measured between the analyzed timing signal and the reference one [1]–[3].

The formulae for the estimators of Allan deviation ADEV and the time deviation TDEV take the form:

$$A\hat{D}EV(t)=\sqrt{\frac{1}{2n^2\tau_0^2(N-2n)}\sum_{i=1}^{N-2n}(x_{i+2n}-2x_{i+n}+x_i)^2} \quad (1)$$

$$T\hat{D}EV(t)=\sqrt{\frac{1}{6n^2(N-3n+1)}\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{j+n-1}(x_{i+2n}-2x_{i+n}+x_i)\right]^2} \quad (2)$$

where $\{x_i\}$ is a sequence of N samples of time error function $x(t)$ taken with interval τ_0; $\tau = n\tau_0$ is an observation interval. In order to simplify the computation process, the formula of the TDEV estimator (2) can be changed [4], [5]. After conversion the formula takes the form:

$$T\hat{D}EV(n\tau_0)=\sqrt{\frac{1}{6}\frac{1}{N-3n+1}\frac{1}{n^2}\sum_{j=1}^{N-3n+1}S_j^2(n)}, \quad (3)$$

where

$$S_j(n)=S_{j-1}(n)-x_{j-1}+3x_{j+n-1}-3x_{j+2n-1}+x_{j+3n-1} \quad (4)$$

and

$$S_1(n)=\sum_{i=1}^{n}(x_{i+2n}-2x_{i+n}+x_i) \quad (5)$$

When computing in the real time, we do not have access to the time error samples indexed by $i+n$ or $i+2n$ for

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a time instant described by index i (the currently measured sample) because these samples have not been measured yet. We have access to the sample currently measured (for the current sampling instant i) and the samples measured earlier (with indexes smaller than i) and stored in the equipment memory. Therefore, the indexes in formulae for ADEV and TDEV estimators must be changed in the case of real-time calculation. The rearrangement of indexes for both estimators was performed in [6]. As a result, we have obtained the ADEV estimator formula for a current instant i in the form depending on the sum of squares of second differences computed for the instant $i-1$:

$$A\hat{D}EV_i(n\tau_0)=\sqrt{K(i,n\tau_0)\Big(A_{i-1}(n)+(x_i-2x_{i-n}+x_{i-2n})^2\Big)} \tag{6}$$

were $K(i,n\tau_0) = 1/(2n^2\tau_0^2(i-2n))$ and $A_i(n)$ is the sum of squares of second differences of time error samples:

$$A_i(n) = \sum_{j=2n+1}^{i} (x_j - 2x_{j-n} + x_{j-2n})^2\,,\ i > 2n \tag{7}$$

The rearrangement of indexes of the time deviation estimator is more complex than in the case of Allan deviation [6]. After changing the indexes of the simplified formula (3-5), we have obtained:

$$T\hat{D}EV_i(n\tau_0) = \sqrt{\frac{1}{6}\frac{1}{i-3n+1}\frac{1}{n^2}S_{ov,i}(n)} \tag{8}$$

where $S_{ov,i}(n)$ is the overall sum updated for each sample i, given in the form:

$$S_{ov,i}(n) = S_{ov,i-1}(n) + S_i^2(n) \tag{9}$$

where

$$S_i(n) = S_{i-1}(n) - x_{i-3n} + 3x_{i+2n} - 3x_{i+n} + x_i,\ i > 3n \tag{10}$$

and

$$S_{3n}(n) = \sum_{j=2n+1}^{3n} (x_j - 2x_{j-n} + x_{j-2n})\,,\ j > 2n \tag{11}$$

Finally, the operations of the real-time TDEV computation for i-th sampling interval are performed using the formula [6]:

$$T\hat{D}EV_i(n\tau_0)=\sqrt{L(i,n)\left[S_{ov,i-1}(n)+(S_{i-1}(n)+\Delta_i(n))^2\right]} \tag{12}$$

where $L(i,n) = 1/\left(6n^2(i-3n+1)\right)$ and:

$$\Delta_i(n) = x_i - 3x_{i+n} + 3x_{i+2n} - x_{i-3n} \tag{13}$$

As a result of the rearrangement of the parameters formulae, in order to compute both parameters, ADEV and TDEV, for a current sampling instant i and given observation interval $\tau = n\tau_0$, we need the values of appropriate sum $A_{i-1}(n)$, $S_{ov,i-1}(n)$, and $S_{i-1}(n)$, currently measured sample x_i and the samples x_{i-n}, x_{i-2n}, and x_{i-3n} previously measured and stored in the equipment memory.

III. Real-time Computation

The formulae of ADEV and TDEV estimators in the forms given by (6) and (12) allow us to perform the calculation in the real time, during the measurement of time error samples. A general procedure of the real-time quasi-parallel ADEV and TDEV computation for a series of observation intervals is as follows [6]:

1) Measure a new time error sample and store it in a data file.
2) Compute the appropriated differences (for ADEV and TDEV) for a given n (observation interval $\tau = n\tau_0$) using the current sample, and the samples measured n, $2n$ or $3n$ sampling intervals earlier.
3) Update the sum for TDEV and sum of squares for ADEV, and compute the square for TDEV.
4) Compute current averages and their square roots.
5) Execute Steps 2-4 for successive larger observation intervals (larger n).
6) Return to Step 1 (measure a new sample).
7) When the measurement is finished, the values of the parameter estimate for the observation intervals considered are known.

Steps 2-5 can be executed when a sufficient number of time error samples were measured, i.e. $2n+1$ samples for a given n. We can compute the first value of ADEV estimate when the sample no. $2n+1$ has been measured. However, for the TDEV the computation of the internal sum $S_i(n)$, given by (11), only just starts. The first value of TDEV estimate we can compute when the sample no. $3n+1$ has been measured.

An example of the real-time ADEV computation for the three observation intervals – $3\tau_0$, $5\tau_0$, $7\tau_0$ – is presented in Fig. 1. Fifteen samples have been measured until now. Three windows, related with the observation intervals considered, are active. These windows – the operators of second difference – indicate adequate samples engaged for calculating a proper second difference, e.g. the window related with n=3 indicates the samples no. 15, 12, and 9.

The computation of TDEV for the first observation interval $\tau = n\tau_0$ begins when the first $2n+1$ samples are measured – for this instant the first item of internal sum $S_{3n}(n)$ can be computed. The sum $S_{3n}(n)$ is updated until the sample number $3n$ is measured. Starting from this instant, the sum $S_i(n)$ is updated using the samples number $i-3n$, $i-2n$, $i-n$ and i, according to (10), and the overall sum $S_{ov,i}(n)$ is updated according to (9). When the updating for a given n is finished, the conditions for successive (greater) observation intervals are checked, and necessary operations for the intervals are performed.

An example of the real-time TDEV computation for the two observation intervals – $3\tau_0$ and $5\tau_0$ – is presented in Fig. 2. The stage of the process after measurement of the sample number 16 is presented. The overall sum $S_{ov,i}(3)$ is updated using the samples number 10, 13, and 16. The internal sum $S_1(3)$ was computed at the early stages of the process and its operator (second difference operator) is not active now. The internal sum $S_1(5)$ was computed and the overall sum $S_{ov,i}(5)$ is updated using the samples number 1, 6, 11, and 16.

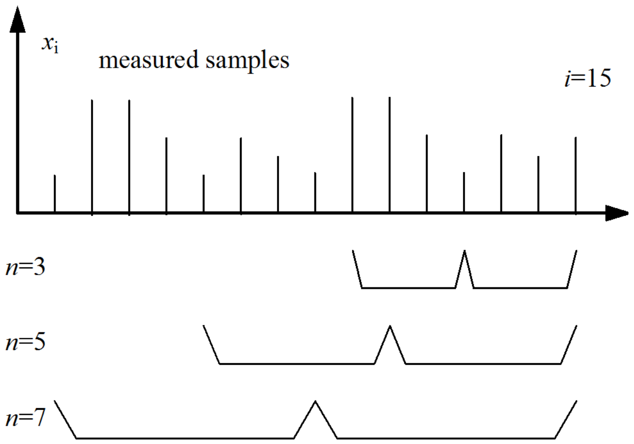

Fig. 1. Real-time ADEV calculation for observation intervals observation intervals $3\tau_0$, $5\tau_0$, $7\tau_0$ sample number 15 is measured.

On-line computation of TDEV is more complex than ADEV computation, especially for the early stages of the process when the internal and overall sums are computed and the computations for some greater observation intervals are not active yet (the conditions of beginning the computations must be checked for each step). In general, in the case of real-time ADEV computation, three samples are involved for a given n: one sample currently measured, and two samples from the past – measured n and $2n$ sampling intervals earlier; in the case of real-time TDEV computation, four samples are involved (besides these three samples, also the sample measured $3n$ sampling intervals earlier) except for the early stages, when the internal sum is updated.

Because the same samples are used for updating the sums in the real-time calculation processes of ADEV and TDEV, we could compute both parameters jointly. The samples needed for computation of both parameters in the current instant can be read out from the equipment memory at once, using one procedure involving three samples (indexed by i, $i-n$, and $i-2n$) at the early stages of the measurement process and four samples (additionally the sample indexed by $i-3n$) at the late stages. Therefore, the influence of the most critical issue – access to the measured data – on the calculation time within one sampling interval can be reduced [7].

An example of the real-time computation of Allan deviation and time deviation for single observation interval $3\tau_0$ performed jointly is presented in Fig. 3 and Fig. 4. The early stage of the process is presented in Fig. 3. This time seven time error samples have been measured until now and the ADEV sum operator and TDEV internal sum operator are active, starting from this instant. The operator of the overall sum $S_{ov}(3)$ is still not active. Fig. 4 presents the stage of the process after the sample no. 10 has been measured. The ADEV operator is active and its sum of squares was updated using samples no. 10, 7, and 4. The TDEV internal operator is not active now – the sum $S(3)$ is computed now and from this instant the overall sum operator (indicating four samples) is active – the first item of the sum $S_{ov}(3)$ can be computed.

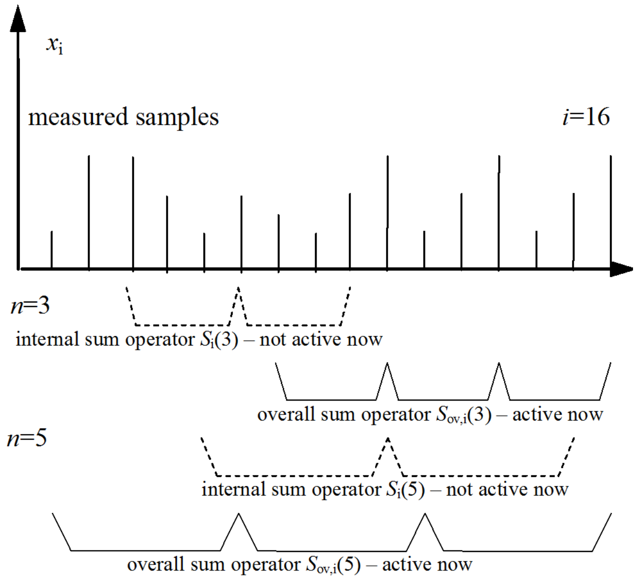

Fig. 2. Real-time TDEV calculation for observation intervals observation intervals $3\tau_0$ and $5\tau_0$, sample number 16 is measured.

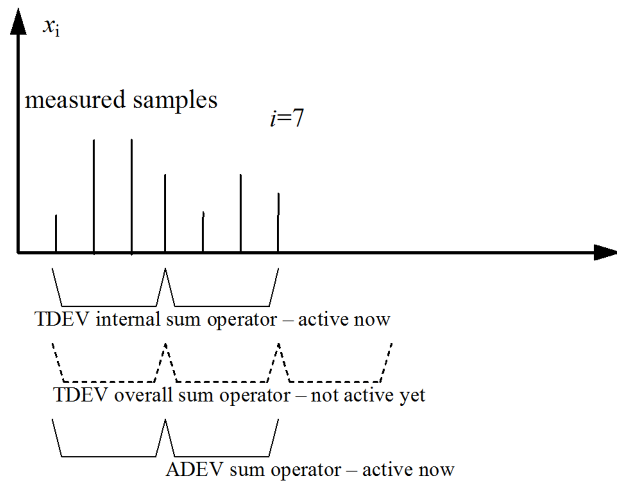

Fig. 3. Joint real-time ADEV and TDEV calculation for observation interval $3\tau_0$, sample number 7 is measured.

IV. Results of Computation Experiment

The methods of separate as well as joint real-time computation of ADEV and TDEV described above were tested in the calculation experiments. The results of the experimental tests were presented in [6], [7]. The calculations were performed off-line but the online work was imitated. The data sequence used in the experiment contains time error samples taken with the sampling interval $\tau_0 = 1/30$ s, representing white phase noise.

The calculations were performed for variable numbers of observation intervals, arranged in the logarithmic scale in a range between 0.1 s and 1000 s. The starting (smallest) observation interval was $\tau_{min} = 0.1$ s ($n = 3$). The longest observation interval was changed from 1 s till 1000 s. The calculations were performed for 5, 10, and 20 observation

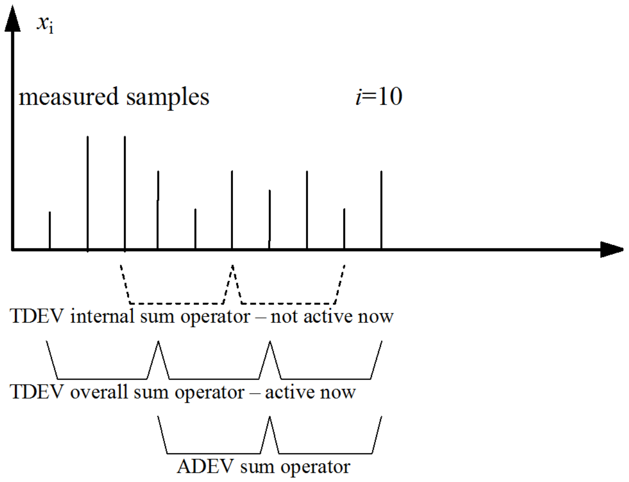

Fig. 4. Joint real-time ADEV and TDEV calculation for observation interval $3\tau_0$, sample number 10 is measured.

TABLE I
TIME OF ADEV CALCULATION

Range of intervals [s]	Number of intervals per decade		
	5	10	20
	t-max [s]	t-max [s]	t-max [s]
0.1-1	0.00012	0.00025	0.0005
0.1-10	0.00024	0.00050	0.0010
0.1-100	0.00034	0.00078	0.0015
0.1-1000	0.00055	0.00110	0.0020

intervals per decade for each range.

The maximum time used for calculation within one sampling interval was the observed quantity. We have assumed that this time cannot exceed the length of sampling interval $\tau_0 = 1/30$ s $= 0.0333\ldots$ s. Personal computer with Intel Pentium IV 3.0 GHz microprocessor was used in the experimental tests.

The time of ADEV computation is presented in TABLE I and the time of TDEV computation is presented in TABLE II [6]. The time of joint ADEV and TDEV computation is presented in TABLE III [7].

The results presented were satisfactory for all cases considered. Even the most time-consuming case – simultaneous computation for 81 observation intervals (the range of τ from 0.1 s till 1000 s and 20 observation intervals for decade) – brought good result. The maximum time of operations performed for one sampling interval does not exceed the sampling interval 1/30 s. Comparing the time of joint TDEV and ADEV computation with the time of TDEV computation, we can see that additional operations of ADEV computation do not influence the maximum time observed for one sampling interval. The comparison of average time of operations performed within one sampling interval for TDEV computation and joint TDEV and ADEV computation presented in [7] confirms the expectation that an additional operation of ADEV computation does not burden the whole process of real-time computation.

TABLE II
TIME OF TDEV CALCULATION

Range of intervals [s]	Number of intervals per decade		
	5	10	20
	t-max [s]	t-max [s]	t-max [s]
0.1-1	0.00018	0.00030	0.00060
0.1-10	0.00030	0.00060	0.00120
0.1-100	0.00050	0.00090	0.00180
0.1-1000	0.00070	0.00130	0.00260

TABLE III
TIME OF TDEV AND ADEV JOINT COMPUTATION

Range of intervals [s]	Number of intervals per decade		
	5	10	20
	t-max [s]	t-max [s]	t-max [s]
0.1-1	0.00018	0.00032	0.00060
0.1-10	0.00030	0.00060	0.00120
0.1-100	0.00050	0.00090	0.00180
0.1-1000	0.00070	0.00130	0.00260

The computation complexity does not depend on the length of observation interval; the number of observation intervals considered is the only limiting factor. Therefore, having limited computational capacities, we can choose wider range of observation intervals or greater number of observation intervals for one decade (resolution of the computation results on the scale of observation intervals). Small number of observation intervals per decade (5 or 10) is sufficient for prompt analysis of timing signal, especially when performing in the real-time. More precise evaluation with the use of greater resolution (greater number of observation intervals) could be performed off-line.

V. CONCLUSIONS

The results of the experimental tests have proved the ability of the real-time computation of Allan deviation and time deviation as well as the real-time computation of both parameters performed jointly. The computation can be performed simultaneously for numerous series and wide range of observation intervals (up to 81 simultaneously analyzed observation intervals were tested). Rather short maximum time spent for computation within one sampling interval allows us to consider joint computation of another additional parameter based on the averaging of second or third difference of time error.

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