Application of Vernier Interpolation for Digital Time Error Measurement

Krzysztof Lange and Michał Kasznia

***Abstract*—The paper discusses potential applications of the time vernier principle, based on the so-called Vernier interpolation. It presents the application of this method to precise time interval measurement and to the results of construction work.**

***Index Terms*—phase detector, time error**

I. INTRODUCTION

TIMING (synchronization) signals in telecommunication networks are affected by a variety of distortion processes, which lower their quality. One of such processes is longterm random phase variation (wander), characterized by a bandwidth below 10 Hz. The basic measure for estimating the quality of timing signal is time error TE, being the difference of phases of the investigated signal and the reference signal, expressed in time units. The precise measurement of TE is of key significance for appropriate estimation of the quality of the timing signal under test.

II. TIME ERROR MEASUREMENT

A typical (standard) technical implementation of TE measurement is the use of a circuit of digital phase detector; its general diagram is shown in Fig. 1. There are two signals, A and B, appended to the detector inputs; their phase difference is the subject of the measurement. The signal coming out from the phase detector is a periodic pattern in which the duration of high state Δt is equal to the phase difference between signals A and B. Precise measurement of the phase difference is then reduced to the accurate measurement of time interval Δt.

The measurement of time interval Δt according to the idea shown in Fig. 1 consists in filling this interval with pulses from a reference generator with frequency f_w, which performs a gate circuit.

Input signals A and B are introduced on the input circuits, which – except for the standardization of the form of these signals – often divide their frequency, reducing it to kilohertz values. This operation is favorable because the extension of duration of the examined interval Δt is proportional to the division ratio , which enables the measurement range to be increased already at this stage of measurement. Unfortunately,

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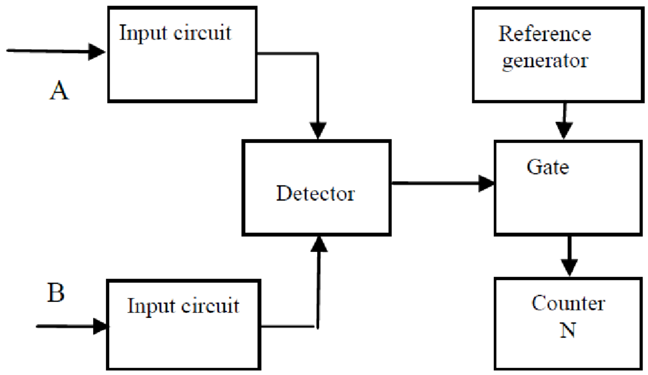

Fig. 1. diagram of a standard phase detector.

applying large values of the division ratio of input signals makes the period T proportionally increased, which extends the time between particular time intervals Δt, and, consequently, it significantly limits the measurement dynamics. The counter, however, determines the number of pulses passing through a gate in time Δt. The counted number N is certainly also proportional to reference frequency f_w, and the standard generator period determines the phase detector resolution, equal to $1/f_w$. To obtain high precision in measuring the phase difference of signals A and B, a high value of reference generator frequency is required. This requirement encounters two troublesome barriers. The underlying cause of the first barrier is the standard frequency generator itself. The higher its frequency, the higher should be the multiplication factor of source signal, which is usually generated by the quartz oscillator. A high multiplication factor increases the phase noise of this signal, which may cause errors greater than the error of insufficient resolution. The other barrier results from the technology of counting pulses by the counter. To obtain the expected high resolution, it is necessary to use digital counters with capacities of several dozen bits, in which first stages must operate correctly with gigahertz frequencies. It leads to emitting huge amounts of heat in them and significantly raises the costs of this solution.

III. TIME VERNIER METHOD

The Vernier interpolation is commonly applied in the form of vernier (i.e. nonius) [1], [2] (in honor of a XV-century mathematician) to precisely measure lengths in two devices: micrometer screw and slide caliper. In each of those devices, depending on the length of the applied base, it is possible to increase the measurement resolution from 10 to 100 or more times. This method is adopted to the digital measurement of time interval by the implementation of a time vernier device with two or three generators. Both circuits operate in similar way as slide caliper. They have two scales: the main scale and vernier scale. These scales have different densities, i.e. – for time interval measurement – different periods of their generators. Respective timing diagrams for methods with 3 and with 2 generators are shown in Fig. 2 and Fig. 3 [1].

The vernier circuit with 3 generators needs a precise reference generator with period T_0 and two quick-start auxiliary generators T_1 and T_2 with periods equal to one another but different from the period of generator T_0. The analysis of the diagram in Fig. 2 allows us to determine a relation between the periods of particular generators and the number of counted pulses in the method with 3 generators, which is shown in (1).

$$\begin{aligned}\Delta t &= T_1 + T_3 - T_2 \\ &= n_1 T_0 \left(1+\frac{1}{n}\right) + n_0 T_0 - n_2 T_0 \left(1+\frac{1}{n}\right) \\ \Delta t &= T_0 \left[n_0 + (n_1 - n_2)\left(1+\frac{1}{n}\right)\right] \quad (1)\end{aligned}$$

Generator T_1 starts at the instance of the beginning of examined interval Δt. Generator T_2 starts at the instance of the end of this interval. Values n with appropriate index denote the number of pulses counted by counters between the time coincidences of pulses from generators T_0 and T_1 as well as T_0 and T_2. Without the vernier circuit interval Δt would be determined according to the formula:

$$\Delta t = T_0 n_0 \quad (2)$$

where n_0 is the number of pulses counted by the counter.

Expression (2) is – as we can easily notice – a fragment of equation (1). A measure of advantage resulting from the application of the vernier method is additional term of equation (1). This is shown in the following expression:

$$\Delta t' = T_0 \left[(n_1 - n_2)\,(\frac{n+1}{n})\right] \quad (3)$$

where n_1 and n_2 are the numbers of pulses counted by respective counters, and n is a coefficient between periods T_0, T_1, T_2, which is shown in formulae:

$$T_1 = T_2 = T_0 \left(\frac{1}{n} + 1\right) \quad (4)$$

$$n = \frac{T_0}{T_1 - T_0} \quad (5)$$

From equation (5) it results that $T_1 = T_2 > T_0$, which means that the frequency of auxiliary generators must be lower than the frequency of standard generator.

The resolution of method with 3 generators is a result of the period of standard generator and coefficient n, which is shown in dependence:

$$\tau = \frac{T_0}{n} \quad (6)$$

A solution of the vernier circuit with 3 generators was proposed by Hewlett Packard in 1980 in a frequency counter. This method is effective because it makes it possible to obtain a resolution at 20 ps level; its practical realization, however, is troublesome. Construction difficulties result from a need to structure two generators with simultaneous quick start, without delay in the trigger pulse, with frequencies equal to one another and fixed frequency relation with the third generator.

A certain simplification is the solution with two generators. In order to preserve the vernier idea, the generators are in mutual frequency relation, which is expressed by a fractional number, similarly as that of equations (4, 5). The elimination of one generator from this solution makes the circuit operation easier because it is easier to design two generators with mutually fixed frequency difference than three such generators. We should remember at the same time that we cannot use the quartz resonator when constructing such generator due to its very high quality factor (with values of order 105 – 106). That is the reason for a considerable time delay at the instance of its start (of millisecond order) – it does not fulfill the assumption of rapid start [1].

The operating principle of the vernier method with 2 generators is shown in a timing diagram in Fig. 3 [3].

The operation of the measurement system of the examined time interval T_x begins at the instance of appearing of the edge that triggers the beginning of the examined interval. Then the generator with time T_1 starts. After the time duration of examined interval ends, the other generator with the duration T_2 is triggered. Both generators produce their signals so long as a time coincidence occurs between the pulses of these generators. Up to this moment, each generator will produce the numbers of pulses, respectively n_1 and n_2. This leads to the following relation:

$$\Delta t = n_1 \cdot T_1 - n_2 \cdot T_2 = (n_1 - n_2) T_1 + n_2 \tau \quad (7)$$

where $\tau = T_1 - T_2$ is the difference of the generator periods, which at the same time expresses the resolution of the method. The fundamental difficulty in the realization of the method is a problem similar to that of the vernier with 3 generators – it is difficult to construct a quick-start generator with good stability parameters. As mentioned above, quartz oscillators cannot be used for that purpose due to their long start, and the keying of these generators causes a random error related with asynchronism between the generator trigger pulse and the generator period. Apart from the difficulties with manufacturing the quick-start generator, there are other difficulties with practical implementation of this method. They concern different problems, and an attempt to minimize their effects unfortunately limits the possibilities to achieve good measurement parameters.

These limitations result from the very idea of the vernier circuit. We can easily notice that the coincidence of signals from two generators will never appear when their output frequencies are equal (to one another). A natural relation appears, therefore, between the time of achieving the coincidence and the measurement resolution, which is determined by the difference of the periods of considered generators, according to equation (7). Also the frequency of quick-start generators influences the coincidence time, which is finally shown in an

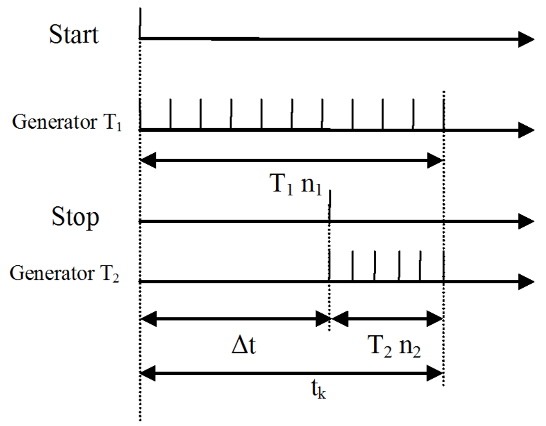

Fig. 2. Timing diagram of vernier with 2 generators.

TABLE I
RELATION BETWEEB COINCIDENCE TIME AND MEASUREMENT RESOLUTION

Resolution τ [ps]	1	5	10	50	100	500
Coincidence time t_k [μs]	400	80	40	8	4	0,8

intuitive dependence:

$$t_k = \frac{T_1 T_2}{T_1 - T_2} \approx \frac{1}{f^2 \tau} + \Delta t \tag{8}$$

where t_k is the time of achieving coincidence, f is an average frequency of generators, Δt is the duration of examined time interval, and τ is the measurement resolution.

The dependence expressing this relation – without taking into account the impact of Δt as well as for an assumed f average value of frequency of the vernier generators 50 MHz and a few possible settings of resolution – is presented in TABLE I. From the relations it results that in order to achieve a low resolution τ of the measurement of time interval Δt, the vernier circuit requires a processing time that can significantly fulfill the inequality:

$$t_k > \Delta t \tag{9}$$

Conclusion of inequality (9) limits the measurement dynamics, which implies that each considered interval must appear at the input of the vernier circuit in time interval greater than time t_k [4].

Technological limitations are result of the potential of implementing the quick-start generators. As already mentioned, it is impossible to introduce into a generator a resonance system with large quality factor which will ensure a frequency instability sufficient during the measurement.

In this situation a possible solution is e.g. to use a generator with delay line, in which the vibration period of this generator will be a function of delay time. Unfortunately, the performance of generator of that type is not stable enough and its output frequency depends, among others, on: temperature, the repeatability of applied circuits, supply voltage or the stability of the delay line itself. Summarizing, technological problems can be, to some degree, reduced to "infecting" a digital system with a quasianalog unit with all consequences of such move.

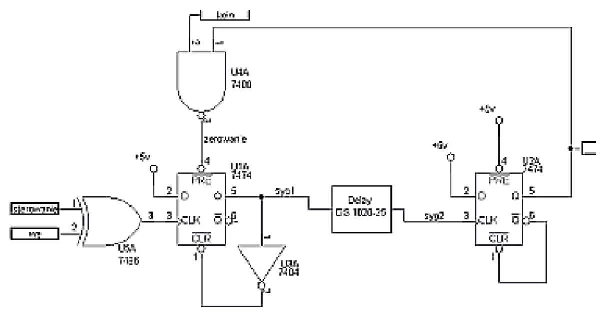

Fig. 3. Schematic diagram of generator.

IV. DESCRIPTION OF CIRCUIT CONSTRUCTION

In the experiment carried out the real signal from the real phase detector was replaced with a precise generator of time interval. It is unimportant from the functional point of view; such replacement, however, makes it easier to set different time intervals, which was proved to be suitable in the analysis of errors of manufactured vernier circuits. The research carried out on the vernier circuit concerns the solution with 2 generators. The most important circuit in this case is the quick-start generator. The authors decided to introduce programmable delay circuits to the generator. Thanks to the control of delay values, attempts to optimize this solution were possible. A schematic diagram of the generator is given in Fig. 4 [4].

The generator consists of two flip-flops 74AC74, delay line Dallas DS1020-25 [5], and gates 74AC00, 74AC86 and 74AC04. A XOR gate is fed with the examined signal. Because there are two such generators in the vernier circuit, the task of the gate is to compensate delays related to the start of the generators, as well as to determine whether a generator should be triggered by the leading (rising) or trailing (falling) edge of the input signal. The operation of the circuit utilizes the idea of positive feedback with the delay line.

The construction of the line requires that the pulse duration to be delayed is longer than the delay time. This is the reason for introducing a negator into the reset circuit of flip-flop U1. The digital line DS1020-25 [4] applied in the circuit has the delay programmed with 8-bit word with step 150 ps. Only 4 lower bits are used, however, because a delay longer than 12,4 ns is not required. This value results from summing up 16x150 ps and 10 ns. 10 ns is the minimum value of the delay that can be achieved in the digital line DS1020-15.

The period of such generator is a sum of delays of particular elements forming the generator. After the digital word is provided at the output, its edge is propagated through flip-flop U1A with time τ_{U1}. Then, the edge is delayed by the digital delay line DS1020 with controllable delay τ_{DS}, passes again onto the flip-flop – this time U2A with delay τ_{U2} – and is sent at the generator output. To pulses generated after the start pulse we should add also the propagation time τ_{NAND} from the output to gate NAND to flip-flop U1A. Therefore, the period of generated measurement signal equals:

$$T = \tau_{U1} + \tau_{U2} + \tau_{NAND} + \tau_{DS} \tag{10}$$

In formula (10) the first three factors are almost constant.

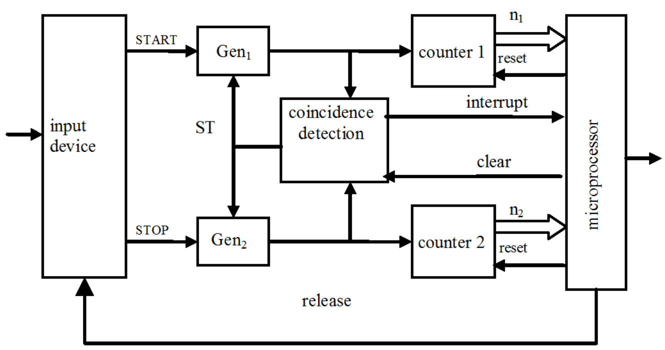

Fig. 4. Vernier block diagram.

They depend exclusively on supply voltage and temperature. Simultaneous control over the period length and frequency is realized by the last factor.

The remaining part of the system is a technical realization of expression (7). It is presented in a block diagram in Fig. 5.

The examined measurement signal is sent at the input circuit which generated signals START and STOP at its input. The time unit between these signals is directly proportional to the examined interval.

From equation (10) we can calculate that the shortest possible period that can be achieved is ca 18,5 ns, which gives a frequency approximately equal to 54 MHz. When the coincidence of pulses from both generators appears, the detection system generates a coincidence signal ST stopping the work of both generators. During the whole operation the systems of counters count pulses from both generators n_1 and n_2. After reading, a microprocessor makes the calculations of the examined interval Δt, resets the counters, and then grants another measurement. The measurement result is displayed on the monitor of a computer cooperating with the microprocessor.

Thanks to the application of two generators which contain independent delay systems DS1020-15, it is possible to choose an appropriate value of τ, dependent only on the difference of words programming the delay systems. Assuming a too small difference, e.g. 50 ps, causes an instability of generators because the instability comprises that difference.

Assuming a too big value of τ places the application of this method under the question mark because the improvement of resolution is very slight. In the manufactured model the value of τ 200 ps was assumed, which is an equivalent to the frequency of reference generator with value 5 GHz.

V. Summary

The developing of the vernier circuit enabled the estimation of its performance. The fundamental objective has been achieved, i.e. the testing of the vernier method and its optimization in the frame of the technology applied. The tests have answered the following questions: which elements of the slotted line are responsible for processing errors, and which points of the system corrections should be introduced. The main task at present is to reduce the manufactured device to FPGA technology, which will eliminate the trouble of generators of the vernier itself and improve their parameters; a decrease in resolution is especially desired. The purpose of further effort is to achieve a resolution level of 20 ps.

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