Scheduling and Capacity Estimation in LTE

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***Abstract*—Due to the variation of radio condition in LTE the obtainable bitrate for active users will vary. The two most important factors for the radio conditions are fading and pathloss. By considering analytical analysis of the LTE conditions including both fast fading and shadowing and attenuation due to distance we have developed a model to investigate obtainable bitrates for customers randomly located in a cell. In addition we estimate the total cell throughput/capacity by taking the scheduling into account. The cell throughput is investigated for three types of scheduling algorithms; Max SINR, Round Robin and Proportional Fair where also fairness among users is part of the analysis. In addition models for cell throughput/capacity for a mix of Guaranteed Bit Rate (GBR) and Non-GBR greedy users are derived.**

Numerical examples show that multi-user gain is large for the Max-SINR algorithm, but also the Proportional Fair algorithm gives relative large gain relative to plain Round Robin. The Max-SINR has the weakness that it is highly unfair when it comes to capacity distribution among users. Further, the model visualize that use of GBR for high rates will cause problems in LTE due to the high demand for radio resources for users with low SINR, at cell edge. Persistent GBR allocation will be a waste of capacity unless for very thin streams like VoIP. For non-persistent GBR allocation the allowed guaranteed rate should be limited.

***Index Terms*—LTE, scheduling, capacity estimation, GBR.**

I. INTRODUCTION

THE LTE (Long Term Evolution) standardized by 3GPP is becoming the most important radio access technique for providing mobile broadband to the mass marked. The introduction of LTE will bring significant enhancements compared to HSPA (High Speed Packet Access) in terms of spectrum efficiency, peak data rate and latency. Important features of LTE are MIMO (Multiple Input Multiple Output), higher order modulation for uplink and downlink, improvements of layer 2 protocols, and continuous packet connectivity [1].

While HSPA mainly is optimized data transport, leaving the voice services for the legacy CS (Circuit Switched) domain, LTE is intended to carry both real time services like VoIP in addition to traditional data services. The mix of both real time and non real time traffic in a single access network requires specific attention where the main goal is to maximize cell throughput while maintaining QoS and fairness both for users and services. Therefore radio resource management will be a key part of modern wireless networks. With the introduction of these mobile technologies, the demand for efficient resource management schemes has increased.

The first issue in this paper is to consider the bandwidth efficiency for a single user in cell for the basic unit of radio resources, i.e. for a RB (Resource Block) in LTE. Since LTE uses advanced coding like QPSK, 16QAM, and 64QAM, the obtainable data rate for users will vary accordingly depending on the current radio conditions. The average, higher moments and distribution of the obtainable data rate for a user either located at a given distance or randomly located in a cell, will give valuable information of the expected cell performance. To find the obtainable bitrate we chose a truncated and downscaled version on Shannon formula which is in line with what is expected from real implementations and also comply with the fact that the maximal bitrate per frequency or symbol for 64 QAM is at most 6 [2].

For the bandwidth efficiency, where we only consider a single user, the scheduling is without any significance. This is not the case when several users are competing for the available radio resources. The scheduling algorithms studied in this paper are those only depending on the radio conditions, i.e. opportunistic scheduling where the scheduled user determined by a given metrics which depends on the SINR (signal-to-interference-plus-noise ratio). The most commonly known opportunistic scheduling algorithms are of this type like PF (Proportional Fair), RR (Round Robin) and Max-SINR. The methodology developed will, however, will apply for general scheduling algorithms where the scheduling metrics for a user is given by a known function of SINR, however, now the SINR may vary in different scheduling intervals taking rapid fading into account. The cell capacity distribution is found for cases where the locations of the users all are known or as an average where all the users are randomly located in the cell [3].

Also the multi user gain (relative increase in cell throughput) due to the scheduling is of main interest. The proposed models demonstrate the magnitude of this gain. As for Max SINR algorithm this gain is expected to be huge, however, the gain comes always at a cost of fairness among users. And therefore fairness has to be taken into account when evaluating the performance of scheduling algorithms.

It is likely that LTE will carry both real time traffic and elastic traffic. We also analyze scenarios where a cell is loaded by two traffic types; high priority CBR (Constant Bit Rate) traffic that requires a fixed data-rate and low priority (greedy) data sources that always consume the leftover capacity not used by the CBR traffic. This is actual a very realistic traffic scenario for future LTE networks where we will have a mix of both real time traffic like VoIP and data traffic. We analyse this case by first estimate the RB usage of the high priority CBR traffic, and then subtract the corresponding RBs to find the actual numbers of RBs available for the (greedy) data traffic sources. Finally we then estimate the cell capacity as the sum of the bitrates offered to the CBR and (greedy) data sources.

The remainder of this paper is organized as follows. In section II the basic radio model is given and models for bandwidth efficiency are discussed. Section III gives an outline of the multiuser case where resource allocation and scheduling

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is taking into account. Some numerical examples are given in section IV and in section V some conclusions are given.

II. Spectrum efficiency

A. *Obtainable bitrate per symbol rate as function of SINR*

For LTE the obtainable bitrate per symbol rate will depend on the radio signal quality (both for up-and downlink). The actual radio signal quality is signaled over the radio interface by the so-called CQI (Channel Quality Indicator) index in the range 1 to 15. Based on the CQI value the coding rate is determined on basis of the modulation QPSK, 16QAM, 64QAM and the amount of redundancy included. The corresponding bitrate per bandwidth is standardized by 3GPP [4] and is shown in Table 1 below. For analytical modeling the actual CQI measurement procedures are difficult to incorporate into the analysis due to the time lag, i.e. the signaled CQI is based measurements taken in earlier TTIs (Transmission Time Interval). To simplify the analyses, we assume that this time lag is set to zero and that the CQI is given as a function of the momentary SINR, i.e. CQI=CQI(SINR). This approximation is justified if the time variation in SINR is significantly slower than the length of a TTI interval. Hence, by applying the CQI table found in [4] we get the obtainable bitrate per bandwidth as function of the SINR as the step function:

$$B = fc_j, \text{ for SINR} \in [g_j, g_{j+1})\,; j = 0, 1, ..., 15, \quad (1)$$

where f is the bandwidth of the channel, c_j is the efficiency for QCI equal j (as given by Table 1) and $[g_j, g_{j+1})$ are the corresponding intervals of SINR values. (We also take $c_0 = 0$, $g_0 = 0$ and $g_{16} = \infty$.)

To fully describe the bitrate function above we also have to also specify the intervals $[g_j, g_{j+1})$. Several simulation studies e.g. [5] suggest that there is a linear relation between the CQI index and the actual SINR limits in [dB]. With this assumption we have $\text{SINR}_j\,[dB] = 10\log_{10} g_j = aj + b$ or $g_j = 10^{\frac{aj+b}{10}}$ for some constants a and b. It is also argued that the actual range of the SINR limits in [dB] is determined by the following (end point) observations: SINR[dB]=-6 corresponds to QCI=1, while SINR[dB]=20 corresponds to CQI=15. Hence we then have $-6 = a + b$ and $20 = 15a + b$ or $a = 13/7$ and $b = -55/7$.

For extensive analytical modelling the step based bandwidth function is cumbersome to apply. An absolute upper bound yields the Shannon formula $B = f\log_2(1+SINR)$, however, we know that the Shannon upper limit is too optimistic. First of all the bandwidth function should never exceed the highest rate $c_{15} = 5.5547$. We therefore suggest downscaling and truncating the Shannon formula and take an alternative bandwidth function as:

$$B = d\min[T, \ln(1 + \gamma\text{SINR})], \quad (2)$$

with $d = f\frac{C}{\ln 2}$ and $T = \frac{c_{15}\ln 2}{C}$ where C is the downscaling constant (relative to the Shannon formula) and γ is a constant less than unity. By choosing C and γ that minimize the square distances between the CQI based and the truncated Shannon formula (2) above we find $C = 0.9449$ and $\gamma = 0.4852$. (Upper and lower estimates of the CQI based zigzagging bitrate function is obtained by taking $\gamma_u = \gamma 10^{a/20} = 0.6008$ and $\gamma_l = \gamma 10^{-a/20} = 0.3918$).

We observe that a downscaling of the Shannon limit is very much in line with the corresponding bitrates obtained by the CQI table as shown in Figure 1 and hence we believe that (2) yields a quite accurate approximation. In fact the approximated CQI values c_j^{app} follow the similar logarithmic behaviour:

$$c_j^{app} = C\log_2(1 + \alpha\beta^j), \quad (3)$$

where now have $\alpha = \gamma 10^{a/20+b/10} = 0.0984$ and $\beta = 10^{a/10} = 1.5336$.

TABLE I
Table 1 CQI table.

CQI index	modulation	code rate x 1024	efficiency
0	out of range		
1	QPSK	78	0.1523
2	QPSK	120	0.2344
3	QPSK	193	0.3770
4	QPSK	308	0.6016
5	QPSK	449	0.8770
6	QPSK	602	1.1758
7	16QAM	378	1.4766
8	16QAM	490	1.9141
9	16QAM	616	2.4063
10	64QAM	466	2.7305
11	64QAM	567	3.3223
12	64QAM	666	3.9023
13	64QAM	772	4.5234
14	64QAM	873	5.1152
15	64QAM	948	5.5547

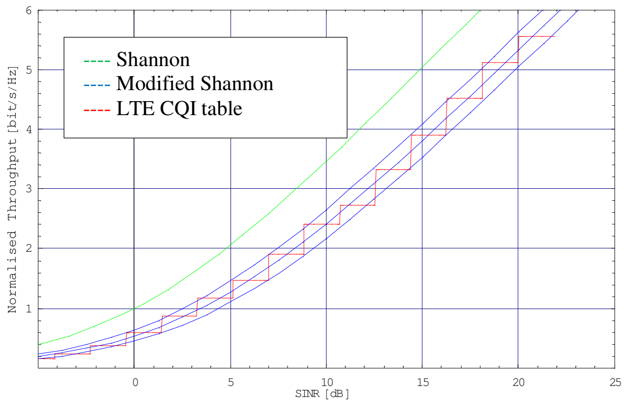

Fig. 1. Normalized throughput as function of the SINR based on: 1.-QCI table, 2.-Shannon and 3.-Modified Shannon.

B. *Radio channel models*

Generally, the SINR for a user will be the ratio of the received signal strength divided by the corresponding noise. The received signal strength is the product of the power P_w times path loss G and divided by the noise component N, i.e. $\text{SINR} = \frac{P_w G}{N}$. Now the path loss G will typical be a stochastic variable depending on physical characteristics such as rapid and slow fading, but will also have a component that are dependent on distance (and possible also the sending frequency). Hence, we first consider variations that are slowly varying over time intervals that are relative long compared with the TTIs (Transmission Time Intervals). Then the path loss is usually given in dB on the form:

$$G = 10^{L/10} \text{ with } L = C - A\log_{10}(r) + X_t, \qquad (4)$$

where C and A are constants, A typical in the range 20-40, and X_t a normal stochastic process with zero mean representing the shadowing (slow fading). The other important component determining the SINR is the noise. It is common to split the noise power into two terms: $N = N_{int} + N_{ext}$ where N_{int} is the internal (or own-cell) noise power and N_{ext} is the external (or other-cell) interference. In a CDMA (Code Division Multiple Access) network, the lack of orthogonality induces own-cell interference. In an OFDMA (Orthogonal Frequency Division Multiple Access) network, however, there is a perfect orthogonality between users and therefore the only contribution to N_{int} is the terminal noise at the receiver. The interference from other cells depends on the location of surrounding base stations and will typically be largest at cell edges. In the following we shall assume that the external noise is constant throughout the cell or negligible, i.e. we assume the noise N to be constant throughout the cell.

Hence, with the assumptions above, we may write SINR on the form $S_t/h(r,\lambda)$ where S_t represent the stochastic variations which we assume to be distance independent capturing the slowly varying fading, and $h(r,\lambda)$ represent the distance dependant attenuation (which we also allow to depend on the sending frequency). Most commonly used channel models as described above have attenuation that follows a power law, i.e. we chose to take $h(r,\lambda)$ on the form

$$h(r,\lambda) = h(\lambda)r^{\alpha}, \qquad (5)$$

where $\alpha = A/10$ is typical in the range 2-4 and $h(\lambda) = \frac{N}{P_w}10^{-C/10}$ with $Z = 10\log_{10}(N) - 10\log_{10}(P_w) - C$ given dB, where we also indicate that $h(\lambda)$ may depend of the (sending) frequency. With the description above the stochastic variable $S_t = 10^{X_t/10}$ with $S_t^{\bullet} = \frac{\ln 10}{10}X_t$, and hence S_t is a lognormal process with $E[S_t^{\bullet}] = 0$ and $\sigma = \frac{\ln 10}{10}\sigma(X_t)$ where $\sigma(X_t)$ is the standard deviation (given in dB) for the normal process X_t. With these assumptions we have the Probability Density Function (PDF) and Complementary Distribution Function (PDF) of S_t as:

$$s_{\ln}(x) = \frac{1}{\sqrt{2\pi}\sigma x}e^{-\frac{(\ln x)^2}{2\sigma^2}} \text{ and } \tilde{S}_{\ln}(x) = \frac{1}{2}erfc\left(\frac{\ln x}{\sigma\sqrt{2}}\right), \qquad (6)$$

where $erfc(y) = \frac{2}{\sqrt{\pi}}\int_{x=y}^{\infty} e^{-x^2}dx$ is the complementary error function.

C. Including fast fading

There are several models for (fast) fading in the literature like Rician fading and Rayleigh fading [6]. In this paper we restrict ourselves to the latter mainly because of its simple negative exponential distribution.

It is possible to include fast fading into the description above. To do so we assume that the fast fading effects are on a much more rapid time scales than slow fading. We therefore assume that the slow fading actual is constant during the rapid fading variations. Hence, condition on the slow fading to be y then for a Rayleigh faded channel the SINR will be exponentially distributed with mean $y/g(r,\lambda)$ Hence, we may therefore take SINR as $S_t/g(r,\lambda)$ where $S_t = X_{\ln}X_e$ is the product of a Log-normal and a negative exponential distributed variables. The corresponding distribution often called Suzuki distribution have PDF ad CDF given as the integrals:

$$s_{su}(x) = \int_{t=0}^{\infty} \frac{1}{t}e^{-\frac{x}{t}}s_{\ln}(t)dt \text{ and } \tilde{S}_{su}(x) = \int_{t=0}^{\infty} e^{-\frac{x}{t}}s_{\ln}(t)dt, \qquad (7)$$

where $s_{\ln}(t)$ is the lognormal PDF above by (6). Since $s_{\ln}(\frac{1}{t}) = t^2 s_{\ln}(t)$ it is possible to express the integrals above in terms of the Laplace transform of the Log-normal distribution and therefore the CDF (and PDF) of the Suzuki distribution may be written as: $\tilde{S}_{su}(x) = \hat{S}_{\ln}(x)$ and $s_{su}(x) = -\hat{S}'_{\ln}(x)$ where

$$\hat{S}_{\ln}(x) = \int_{t=0}^{\infty} e^{-xt}s_{\ln}(t)dt = \frac{1}{\sqrt{2\pi}\sigma}\int_{t=0}^{\infty}\frac{e^{-t-\frac{(\ln(t/x))^2}{2\sigma^2}}}{t}dt \qquad (8)$$

is the Laplace transform of the Log-normal distribution. If we define the truncated transform:

$$\begin{aligned}\tilde{S}_{su}(x,M) &= \frac{1}{x}\int_{t=0}^{M} e^{-t}s_{\ln}(t/x)dt \\ &= \frac{1}{\sqrt{2\pi}\sigma}\int_{t=0}^{M}\frac{e^{-t-\frac{(\ln(t/x))^2}{2\sigma^2}}}{t}dt, \qquad (9)\end{aligned}$$

then $\tilde{S}_{su}(x) = \lim_{M\to\infty}\tilde{S}_{su}(x,M)$ and further the corresponding error is exponentially small. An attempt to expand the integral (8) in terms of the series of the exponential function $e^{-t} = \sum_{k=0}^{\infty}\frac{(-1)^k t^k}{k!}$ yields a divergent series; however, this is not the case for the truncated transform (9). We find the following series expansion:

$$\tilde{S}_{su}(x,M) = \frac{1}{2}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}x^k e^{\frac{k^2\sigma^2}{2}} erfc\left(\frac{k\sigma}{\sqrt{2}} + \frac{\ln(x/M)}{\sigma\sqrt{2}}\right) \qquad (10)$$

Similar the PDF of the Suzuki random variable may be found from (8) by differentiation:

$$\begin{aligned}s_{su}(x) = -\tilde{S}'_{su}(x) &= \int_{t=0}^{\infty} e^{-xt}ts_{\ln}(t)dt \\ &= \frac{1}{\sqrt{2\pi}\sigma x}\int_{t=0}^{\infty} e^{-t-\frac{(\ln(t/x))^2}{2\sigma^2}}dt, \qquad (11)\end{aligned}$$

and for the PDF we now we take the corresponding truncated integral to be:

$$s_{su}(x,M) = \frac{1}{\sqrt{2\pi}\sigma x}\int_{t=0}^{M} e^{-t-\frac{(\ln(t/x))^2}{2\sigma^2}}dt \qquad (12)$$

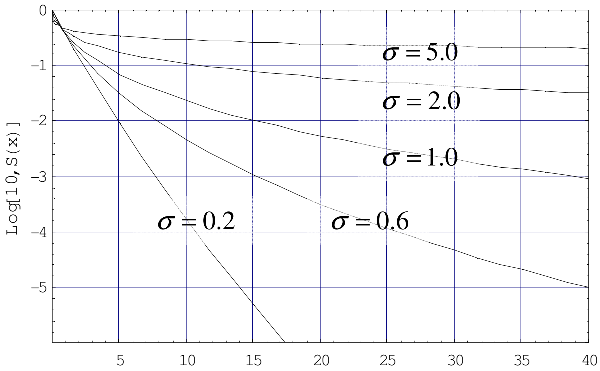

Fig. 2. Logarithmic plot of the CDF for the Suzuki distribution as function of x for some values of σ.

In this case we find $0 \leq s_{su}(x) - s_{su}(x, M) = e^{-M+\frac{\sigma^2}{2}}$ as a bound of the truncation error.

By expanding the integral (12) in terms of the exponential function as above, we now obtain a similar (convergent) series:

$$s_{su}(x, M) = \frac{1}{2}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}x^k e^{\frac{(k+1)^2\sigma^2}{2}} erfc\left(\frac{(k+1)\sigma}{\sqrt{2}} + \frac{\ln(x/M)}{\sigma\sqrt{2}}\right) \tag{13}$$

In Figure 2 we have plotted the CDF of the Suzuki distribution for σ equals 0.2, 0.6, 1.0, 2.0 and 5.0. (The CDF Suzuki distribution is calculated by applying the series (10) with $M = 20.0$ which secure an accuracy of 2.0x10-9 in the computation.) Note that the k'th moment of the Suzuki distribution is $k!$ that of the Log-normal.

D. Distribution of the obtainable bitrate for channel of a certain bandwidth for a user located at a given distance from the sender antenna

Below we express the distribution of the possible obtainable bitrate according to the distribution of the stochastic part of the SINR; namely S_t. From (1) we get the bit-rate $B_t(r)$ for a channel occupying a bandwidth f located at distance r as:

$$B_t(r) = fc_j \quad \text{when} \quad S_t \in [h(r,\lambda)g_j, h(r,\lambda)g_{j+1}), \quad \text{for} \quad j = 0, 1, ..., 15. \tag{14}$$

Hence, the DF (Distribution Function) of the bandwidth distribution for a user located at distance r; $B(y,r) = P(B_t(r) \leq y)$ may be written:

$$B(y,r) = S(h(r,\lambda)g_{j+1}), \quad \text{for} \quad y \in (fc_j, fc_{j+1}] \quad \text{for} \quad j = 0, 1, ..., 15, \tag{15}$$

where $S(x)$ is the DF of the variable fading component. Hence, we obtain the k'moment of the obtainable bitrate for a user located at a distance r from the antenna as the (finite) sum:

$$m_k(r) = f^k \sum_{j=1}^{15}\left(c_j^k - c_{j-1}^k\right)\tilde{S}(h(r,\lambda)g_j), \tag{16}$$

where $\tilde{S}(x) = 1 - S(x)$ is the CDF of the variable fading component.

Rather than applying the discrete modeling approach above we may prefer to apply the smooth (continuous) counterpart defined by relation (2). The bit-rate $B_t(r)$ for a channel occupying a bandwidth f located at distance r is then given by

$$B_t(r) = d\min[T, \ln(1 + S_t/g(r,\lambda))], \tag{17}$$

with $d = f\frac{C}{\ln 2}$ and $T = \frac{c_{15}\ln 2}{C}$ and where C is the downscaling constant (relative to the Shannon formula) and where we also define $g(r,\lambda) = \gamma^{-1}h(r,\lambda)$. For the continuous bandwidth case the DF of the bandwidth distribution for a user located at distance r is given by:

$$B(y,r) = \begin{cases} S(g(r,\lambda)(e^{y/d} - 1)) & \text{for } y/d < T \\ 1 & \text{for } y/d \geq T \end{cases} \tag{18}$$

Based on (18) we may write the k'moment of the obtainable bitrate for a user located at a distance r from the antenna:

$$\begin{aligned} m_k(r) &= d^k \int_{y=0}^{g(r,\lambda)(e^T-1)} \left(\ln\left(1 + y/g(r,\lambda)\right)\right)^k s(y)dy + \\ &\quad + d^k T^k \tilde{S}(g(r,\lambda)(e^T - 1) \end{aligned} \tag{19}$$

E. Distribution of the obtainable bitrate for channel of a certain bandwidth for a user that is randomly placed in a circular cell with power-law attenuation

Since the bitrate/capacity for a user will strongly depend of the distance from the sender antenna, a better measure of the capacity will be to find the distribution of bitrate for a user that is randomly located in the cell. This is done by averaging over the cell area and therefore the distribution of the corresponding averaging bitrate B_t is given as $B(y) = \frac{1}{A}\int_A B(y,r)dA(r)$ where A is the cell area. For circular cell shape and power law attenuation on the form $h(r,\lambda) = h(\lambda)r^\alpha$ (where we also take $g(\lambda) = \gamma^{-1}h(\lambda)$ i.e. $g(r,\lambda) = g(\lambda)r^\alpha$) the corresponding integral may be partly evaluated. By defining an α-factor averaging variable S_α with DF $S_\alpha(x) = P(S_\alpha \leq x)$ given by

$$S_\alpha(x) = \frac{2}{\alpha}x^{-\frac{2}{\alpha}} \int_{t=0}^{x} t^{\frac{2}{\alpha}-1}S(t)dt = \frac{2}{\alpha}\int_{t=0}^{1} t^{\frac{2}{\alpha}-1}S(tx)dt \tag{20}$$

and with PDF

$$s_\alpha(x) = \frac{2}{\alpha}x^{-\frac{2}{\alpha}-1} \int_{t=0}^{x} t^{\frac{2}{\alpha}}s(t)dt = \frac{2}{\alpha}\int_{t=0}^{1} t^{\frac{2}{\alpha}}s(tx)dt \tag{21}$$

the bitrate distribution will have the exact same form as (15) for the discrete bandwidth case and (18) for the continuous bandwidth case, and with moments given by (16) and (19) by changing $r \rightarrow R$ and $S(x) \rightarrow S_\alpha(x)$ (and $s(x) \rightarrow s_\alpha(x)$).

1) Distribution of the stochastic variable S_α for Log-normal and Suzuki distribution: Based on the definition we may derive the CDF and PDF of stochastic variable S_α for the Log-normal and Suzuki distributed fading models. For the Log-normal distribution we have

$$\tilde{S}_{\ln_\alpha}(x) = \frac{1}{\alpha x^{2/\alpha}} \int_{t=0}^{x} t^{2/\alpha-1} erfc\left(\frac{\ln t}{\sigma\sqrt{2}}\right)dt.$$

By changing variable according to $y = \ln t$ in the integral we find:

$$\tilde{S}_{\ln_\alpha}(x) = \frac{1}{2}\left(erfc\left(\frac{\ln x}{\sigma\sqrt{2}}\right) + \right.$$
$$\left. + x^{-2/\alpha} e^{2\sigma^2/\alpha^2} erfc\left(\frac{2\sigma^2 - \alpha \ln x}{\alpha\sigma\sqrt{2}}\right)\right) \quad (22)$$

and further the PDF is found by differentiation:

$$s_{\ln_\alpha}(x) = \frac{1}{\alpha} x^{-(2/\alpha+1)} e^{2\sigma^2/\alpha^2} erfc\left(\frac{2\sigma^2 - \alpha \ln x}{\alpha\sigma\sqrt{2}}\right) \quad (23)$$

For the Suzuki distribution we have the CDF given by the integral $\tilde{S}_{su}(x) = x\int_{t=0}^{\infty} t^{-2}e^{-t}s_{\ln}(x/t)dt$ and therefore we have:

$$\tilde{S}_{su_\alpha}(x) = \frac{2}{\alpha}\int_{t=0}^{1} t^{2/\alpha-1}\tilde{S}_{su}(xt)dt$$
$$= x\int_{t=0}^{\infty} t^{-2}e^{-t}s_{\ln_\alpha}(x/t)dt \quad (24)$$

where $s_{\ln_\alpha}(x)$ is given by (23) above for the Lognormal distribution. As for the Suzuki distribution approximation to any accuracy is possible to obtain of $\tilde{S}_{su_\alpha}(x)$ by truncating the integral above:

$$\tilde{S}_{su_\alpha}(x, M) = x\int_{t=0}^{M} t^{-2}e^{-t}s_{\ln_\alpha}(x/t)dt \quad (25)$$

and also for this case we find that the truncation error is exponentially small. By expanding $e^{-t} = \sum_{k=0}^{\infty}\frac{(-1)^k t^k}{k!}$ and integrating term by term we find:

$$\tilde{S}_{su_\alpha}(x, M) = \sum_{k=0}^{\infty}\frac{(-1)^k x^k}{(2+k\alpha)k!}e^{\frac{k^2\sigma^2}{2}} erfc\left(\frac{k\sigma}{\sqrt{2}} + \frac{\ln(x/M)}{\sigma\sqrt{2}}\right) +$$
$$+ \frac{e^{2\sigma^2/\alpha^2}}{\alpha}\gamma\left(\frac{2}{\alpha}, M\right)x^{-2/\alpha} erfc\left(\frac{2\sigma^2 - \alpha\ln(x/M)}{\alpha\sigma\sqrt{2}}\right) \quad (26)$$

where $\gamma(a, x) = \int_{t=0}^{x} t^{a-1}e^{-t}dt$ is the incomplete gamma function. (Observe the similarity with the corresponding expansion for $\tilde{S}_{su}(x)$ by (10).)

The corresponding integral for the PDF is given by:

$$s_{su_\alpha}(x) = \int_{t=0}^{\infty} t^{-1}e^{-t}s_{\ln_\alpha}(x/t)dt \quad (27)$$

and we take the truncated approximation of the PDF as the integral:

$$s_{su_\alpha}(x, M) = \int_{t=0}^{M} t^{-1}e^{-t}s_{\ln_\alpha}(x/t)dt \quad (28)$$

and we find the following error bound: $0 \leq s_{su_\alpha}(x) - s_{su_\alpha}(x, M) \leq e^{-M+\frac{\sigma^2}{2}}$. By the similar approach as for the CDF we find the following series expansion of the truncated PDF:

$$s_{su_\alpha}(x, M) =$$
$$= \sum_{k=0}^{\infty}\frac{(-1)^k x^k}{(2+(k+1)\alpha)k!}e^{\frac{(k+1)^2\sigma^2}{2}} erfc\left(\frac{(k+1)\sigma}{\sqrt{2}} + \frac{\ln(\frac{x}{M})}{\sigma\sqrt{2}}\right)$$
$$+ \frac{e^{\frac{2\sigma^2}{\alpha^2}}}{\alpha}\gamma\left(\frac{2}{\alpha}+1, M\right)x^{-(1+\frac{2}{\alpha})} erfc\left(\frac{2\sigma^2 - \alpha\ln(\frac{x}{M})}{\alpha\sigma\sqrt{2}}\right)$$
$$(29)$$

III. ESTIMATION OF CELL CAPACITY

In the following we assume that the cell is loaded by two traffic types:

- High priority CBR traffic sources that each requires to have a fixed data-rate and
- Low priority (greedy) data sources that always consumes the leftover capacity not used by the CBR traffic.

This is actually a very realistic traffic scenario for future LTE networks where we actual will have a mix of both real time traffic like VoIP and typical elastic data traffic. Below, we first estimate the RB usage of the high priority CBR traffic, and then we may subtract the corresponding RBs to find the actual numbers of RBs available for the (greedy) data traffic sources. Then finally we estimate the cell throughput/capacity as the sum of the bitrates offered to the CBR and (greedy) data sources.

A. Estimation of the capacity usage for GBR sources in LTE

The reservation strategy considered simply allocate recourses on a per TTI bases and allocate RBs so that the aggregate rate equals the required GBR (Guaranteed Bit Rate) rate (Non-Persistent scheduling).

1) Capacity usage for a single GBR source : We first consider the case where we know the location of the CBR user in the cell, i.e. at a distance r from the antenna. We take B as the bitrate obtainable for a single RB and consider a GBR source that requires a fixed bit-rate of b^{CBR}. We assumes that this is achieved by offering n RBs for every k-TTI interval. A way of reserving resources to GBR sources is to allocate RBs so that $\frac{n}{k}B$ will be close to the required rate b^{CBR} over a given period. We take $N_{CBR} = \frac{n}{k}$ to be the number of RBs granted to a GBR connection in a TTI as (the stochastic variable):

$$N_{CBR} = \begin{cases} \frac{\alpha b^{CBR}}{B} & \text{if } CQI > 0 \\ 0 & \text{if } CQI = 0 \end{cases}, \quad (30)$$

where we have introduced a scaling factor α so that on the long run we obtain the desired GBR-rate b^{CBR}. By choosing $\alpha = p_{CQI}^{-1}$ where $p_{CQI} = P(CQI > 0) = \tilde{S}(h(r, \lambda)g_1)$ then $E[N_{CBR}B] = b^{CBR}$ and hence we also have:

$$E[N_{CBR}|CQI > 0] = \frac{b^{CBR}}{p_{CQI}}E\left[B^{-1}|CQI > 0\right]. \quad (31)$$

The mean numbers of RBs is therefore:

$$\beta = \beta(r, b^{CBR}) = b^{CBR}m_{-1}^{CQI}(r), \quad (32)$$

where the conditional moments $m_k^{CQI}(r) = E\left[B^k \,|CQI > 0\right]$ is found as

$$m_k^{CQI}(r) = \frac{f^k}{\tilde{S}(h(r,\lambda)g_1)} \left(c_1^k \tilde{S}(h(r,\lambda)g_1) + \right. \\ \left. + \sum_{j=2}^{15} \left(c_j^k - c_{j-1}^k\right) \tilde{S}(h(r,\lambda)g_j) \right), \tag{33}$$

for the discrete bandwidth case and by

$$m_k^{CQI}(r) = \\ \frac{d^k}{\tilde{S}(h(r,\lambda)g_1)} \left(\int\limits_{y=h(r,\lambda)g_1}^{g(r,\lambda)(e^T-1)} \left(\ln\left(1+\frac{y}{g(r,\lambda)}\right)\right)^k s(y)dy + \right. \\ \left. + T^k \tilde{S}(g(r,\lambda)(e^T-1) \right) \tag{34}$$

for the continuous bandwidth case. Note that by conditioning on having $CQI > 0$ we exclude the users that are unable to communicate due to bad radio conditions and avoid the problems due to division of zero in the calculation of the mean of $1/B$.

For circular cells and power law attenuation we obtain the corresponding result as above by changing $r \to R$ and $S(x) \to S_\alpha(x)$.

2) Estimation of RBs usage for several CBR sources: We first estimate the RB usage for a fixed number of M CBR sources located at distances r_j from the antenna and with bit-rate requirements b_j^{CBR} $j = 1, ..., M$. The total usage of RBs β^{CBR} will be the sum the individual contribution from each source as given by (32):

$$\beta^{CBR} = \sum_{j=1}^{M} \beta(r_j, b_j^{CBR}). \tag{35}$$

For the case with random location the expression gets even simpler:

$$\beta^{CBR} = \beta(R, \sum_{j=1}^{M} b_j^{CBR}), \tag{36}$$

i.e. we may add the CBR rates from all the sources in the cell. The corresponding throughput for the CBR sources is taken as the sum of the individual rates i.e.

$$b_{CBR} = \sum_{j=1}^{M} b_j^{CBR} \tag{37}$$

B. Estimation of the capacity usage for a fixed number of greedy sources

We shall estimate the capacity usage for a fixed number of greedy sources under the following assumptions:

- There are totally K active (greedy) users that are placed random in the cell which always have traffic to send, i.e. we consider the cell in saturated conditions.
- There is totally N available RBs and the scheduled user is granted all of them in a TTI interval.

1) Scheduling of based on metrics: In the following we consider the case with K users that are located in a cell with distances from the sender antenna given by a distance vector $\mathbf{r} = (r_1, ..., r_K)$ and we assume that the user scheduled in a TTI is based on:

$$i_{schedul} = \arg \max_{i=1,..,K} \{M_i\}, \tag{38}$$

where $M_i = M_i(\mathbf{r})$ is the scheduling metric which also may depend on the location of all users (through the location vector $\mathbf{r} = (r_1, ..., r_K)$). Hence, for the scheduler to choose user i, the metric M_i must be larger than all the other metrics (for the other users), i.e. we must have $M_i > U_i$ where

$$U_i = \max_{\substack{k=1,..,K \\ k \neq i}} M_k. \tag{39}$$

Since we assume that a user is granted all the RBs when scheduled, this gives the cell throughput when user is scheduled (located at distance r_i) to be $NB(r_i)$, where $B(r_i)$ is the corresponding obtainable bit-rate per RB. Hence, cell bit-rate distribution (with K users located in the cell with distance vector $\mathbf{r} = (r_1, ..., r_K)$) may then be written as:

$$B_g(y, \mathbf{r}) = \sum_{i=1}^{K} B_i(y, \mathbf{r}), \text{ where} \tag{40}$$

$$B_i(y, \mathbf{r}) = P\left(NB(r_i) \leq y, M_i(\mathbf{r}) > U_i(\mathbf{r})\right) \tag{41}$$

is bitrate distribution when user i is scheduled. Unfortunately, for the general case exact expression of the probabilities $B_i(y, \mathbf{r})$ is difficult to obtain mainly due to the involvement of the scheduling metrics. However, for some cases of particular interest closed form analytical expression is possible to obtain. For many scheduling algorithms the scheduling metrics is only function of the SINR for that particular user (and does not depend of the SINR for the other users) and for this case extensive simplification is possible to obtain. In the following we therefore assume that the metrics M_i only are functions of their own SINR$_i$ and the location r_i for that particular user, i.e. we have $M_i = M(S_i, r_i)$, where we (for simplicity) also assume that $M(x, r_i)$ is an increasing function of x with an unlikely defined inverse $M^{-1}(x, r_i)$. The distribution functions for M_i and $U_i = \max_{\substack{k=1,..,K \\ k \neq i}} M_k$ are then

$$M_i(x, r_i) = P(M_i \leq x) = S(M^{-1}(x, r_i)) \text{ and} \tag{42}$$

$$U_i(x, \mathbf{r}) = P(U_i \leq x) = \prod_{k=1, k\neq i}^{K} S(M^{-1}(x, r_k)) \tag{43}$$

If we now condition on the value of $S_i = x$ in (41), we find the distribution of the cell capacity when user i is scheduled as:

$$B_i(y, \mathbf{r}) = \int\limits_{x=0}^{\infty} P\left(B(r_i) \leq \frac{y}{N} \middle| \, S_i = x\right) U_i(M(x, r_i), \mathbf{r}) s(x) dx. \tag{44}$$

By using (14) as the obtainable bit-rate per RB for the discrete case we find:

$$B_i(y,\mathbf{r}) \;=\; \int_{x=0}^{h(r_i,\lambda)g_{j+1}} F_i(x,\mathbf{r})s(x)dx, \text{ if } y/N \in (fc_j, fc_{j+1}] \quad \text{for } j=0,1,...,15, \tag{45}$$

where we now have defined the multiuser "scheduling" function $F_i(x,\mathbf{r})$ by:

$$F_i(x,\mathbf{r}) = U_i(M(x,r_i),\mathbf{r}) = \prod_{k=1,k\neq i}^{K} S(M^{-1}(M(x,r_i),r_k)) \tag{46}$$

Similar for the continuous case based on (17) as the obtainable bit-rate per RB gives:

$$B_i(y,\mathbf{r}) = \begin{cases} \int\limits_{x=0}^{g(r_i,\lambda)(e^{y/dN}-1)} F_i(x,\mathbf{r})s(x)dx & \text{for } y/dN < T \\ p_i(\mathbf{r}) & \text{for } y/dN \geq T \end{cases}, \tag{47}$$

where $p_i(\mathbf{r}) = \int_{x=0}^{\infty} F_i(x,\mathbf{r})s(x)dx$ is the probability that user is scheduled in a TTI (and therefore $\sum_{i=1}^{K} p_i(\mathbf{r}) = 1$).

Finally, by assuming that all users are randomly located throughout the cell the corresponding bit-rate distribution is found by performing a K-dimensional averaging over all possible distance vectors $\mathbf{r}$, over the cell; $B_g(y) = \frac{1}{A^K}\int_A \cdots \int_A r_1 \dots r_K B_{cell}(y,\mathbf{r})dA_1 \cdots dA_K$, where A here is the cell area. Due to the special form of the function $F_i(x,\mathbf{r}) = \prod_{k=1,k\neq i}^{K} S(M^{-1}(M(x,r_i),r_k))$ the "cell averaging" over the $K-1$ dimension variables $r_1,\dots,r_{i-1},r_{i+1},\dots,r_K$ (not including the variable r_i) yields the product $\left[\widehat{S}(M(x,r_i)\right]^{K-1}$ where

$$\widehat{S}(y) = \frac{1}{A}\int_A uS(M^{-1}(y,u))dA(u) \tag{48}$$

Hence, for the case when user i is located at distance r_i and all the $K-1$ other users located at random, then we find for the discrete case:

$$B_i(y,r_i) = \int_{x=0}^{h(r_i,\lambda)g_{j+1}} \left[\widehat{S}(M(x,r_i))\right]^{K-1} s(x)dx, \text{ if } y/N \in (fc_j, fc_{j+1}] \quad \text{for } j=0,1,...,15 \tag{49}$$

and for the continuous case:

$$B_i(y,r_i) = \begin{cases} \int\limits_{x=0}^{g(r_i,\lambda)(e^{y/dN}-1)} \left[\widehat{S}(M(x,r_i))\right]^{K-1} s(x)dx & \text{for } y/dN < T \\ p_i(r_i) & \text{for } y/dN \geq T, \end{cases} \tag{50}$$

where $p_i(r_i) = \int_{x=0}^{\infty}\left[\widehat{S}(M(x,r_i))\right]^{K-1} s(x)dx$ is the probability that user i is scheduled. (Observe that the $p_i(r) = p(r)$ and $B_i(y,r) = B(y,r)$ only depend on the location r_i and hence are equal for all the users.)

For circular cell size the cell bit-rate distribution integrals above is reduced to:

$$B_g(y) \;=\; \frac{2}{R^2}\int_{r=0}^{R} r \int_{x=0}^{h(r,\lambda)g_{j+1}} K\left[\widehat{S}(M(x,r))\right]^{K-1} s(x)dxdr \quad \text{if } y/N \in (fc_j, fc_{j+1}]\,; \text{ for } j=0,1,...,15 \tag{51}$$

for the discrete case and

$$B_g(y) = \begin{cases} \frac{2}{R^2}\int\limits_{r=0}^{R} rL(y,r)dr & \text{for } y/dN < T \\ 1 & \text{for } y/dN \geq T \end{cases} \tag{52}$$

where $L(y,r) = \int_{x=0}^{g(r,\lambda)(e^{y/dN}-1)} K\left[\widehat{S}(M(x,r))\right]^{K-1} s(x)dx$. For the continuous case where we now have

$$\widehat{S}(y) = \frac{2}{R^2}\int_{r=0}^{R} uS(M^{-1}(y,u))du \tag{53}$$

The moments of the capacity (when the users are located according to the vector $\mathbf{r} = (r_1,\dots,r_K)$ may be written as:

$$E[B_g(\mathbf{r})^k] = f^k N^k \sum_{i=1}^{K}\sum_{j=1}^{15} c_j^k \int_{x=h(r_i,\lambda)g_j}^{h(r_i,\lambda)g_{j+1}} F_i(x,\mathbf{r})s(x)dx \tag{54}$$

for the discrete bandwidth case and

$$E[B_g(\mathbf{r})^k] = d^k N^k \sum_{i=1}^{K}\left(\int_{x=0}^{g(r_i,\lambda)(e^T-1)} (\ln(1+x/g(r_i,\lambda)))^k F_i(x,\mathbf{r})s(x)dx + T^k \int_{x=g(r_i,\lambda)(e^T-1)}^{\infty} F_i(x,\mathbf{r})s(x)dx \right), \tag{55}$$

for the continuous case.

The corresponding moments for the case where the users are randomly located in a circular cell are given by:

$$E[B_g^k] = \frac{2f^kN^k}{R^2}\sum_{j=1}^{15} c_j^k \int_{r=0}^{R} r \left\{ \int_{x=h(r,\lambda)g_j}^{h(r,\lambda)g_{j+1}} K\left[\widehat{S}(M(x,r))\right]^{K-1} s(x)dx \right\} dr \tag{56}$$

for the discrete bandwidth case and

$$E[B_g^k] = \frac{2d^kN^k}{R^2}\int_{r=0}^{R} r\left(\int_{x=0}^{g(r,\lambda)(e^T-1)} \left(\ln\left(1+\frac{x}{g(r,\lambda)}\right)\right)^k L(x,r)s(x)dx + T^k \int_{x=g(r,\lambda)(e^T-1)}^{\infty} L(x,r)s(x)dx \right) dr \tag{57}$$

for the continuous case, where $L(x,r) = K\left[\widehat{S}(M(x,r))\right]^{K-1}$.

2) Examples: Below we consider and compare three of the most commonly known scheduling algorithms, namely Round Robin (RR), Proportional Fair (PF) and Max SINR by applying the cell capacity models described above.

a) Round Robin : For the Round Robin algorithm each user is given the same amount of bandwidth and hence this case corresponds to taking $K = 1$ i.e. the results in section II may be applied by to find the cell capacity with $f \to Nf$ and $S(x) \to S_{su}(x)$ and also $S_\alpha(x) \to S_{su_\alpha}(x)$.

b) Proportional Fair (in SINR) : Normally, the shadowing is varying over a much longer time scale than the TTI intervals, and hence we may assume that the slow fading is constant during the updating of the scheduling metric M_i and therefore should only account for the rapid fading component. This means that the shadowing effect may be taken as constant that may be included in the non varying part of the SINR over several TTI intervals. Hence, we take SINR as $S_t/g(r;\lambda)$ where $S_t = zX_e$ conditioned that the shadowing $X_{\ln} = z$. By assuming that $X_{\ln} = z$ is constant over the short TTI intervals the scheduling metrics will be $M_i = \frac{zX_e/h(r_i,\lambda)}{zE[X_e]/h(r_i,\lambda)} = \frac{X_e}{E[X_e]}$. In the final result we then "integrate over the Log-normal slow fading component". We find that the probability of being scheduled is $p(r) = \frac{1}{K}$ and that the conditional bandwidth distribution for a user at located at distance r (and the $K-1$ users random located) is given by the results in section II-D with $f \to Nf$ and $S(x) \to S_K(x)$ with:

$$\begin{aligned} S_K(x) &= \int_{t=0}^{\infty} S_e\left(\frac{x}{t}\right)^K s_{\ln}(t)dt \text{ and} \\ s_K(x) &= \int_{t=0}^{\infty} \frac{K}{t} S_e\left(\frac{x}{t}\right)^{K-1} s_e\left(\frac{x}{t}\right) s_{\ln}(t)dt, \end{aligned} \quad (58)$$

where $S_e(x) = 1 - e^{-x}$ and $s_e(x) = e^{-x}$.

Further, the distribution of the cell capacity is given by the results in section II-E with $f \to Nf$ and further the α-averaging is given by the integrals:

$$\tilde{S}_{K_\alpha}(x) = \int_{t=0}^{\infty} KS_e(t)^{K-1} s_e(t) \tilde{S}_{\ln_\alpha}\left(\frac{x}{t}\right) dt \text{ and} \quad (59)$$

$$s_{K_\alpha}(x) = \int_{t=0}^{\infty} KS_e(t)^{K-1} s_e(t) t^{-1} s_{\ln_\alpha}\left(\frac{x}{t}\right) dt \quad (60)$$

c) Max SINR algorithm.: For this algorithm the scheduling metric is $M_i = S_i/h(r_i,\lambda)$. By assuming circular cell size and radio signal attenuation on the form $h(r,\lambda) = h(\lambda)r^\alpha$ gives:

$$\widehat{S}(M(x,r)) = \frac{2}{R^2} \int_{r=0}^{R} uS(x(u/r)^\alpha)du = S_\alpha(x(R/r)^\alpha). \quad (61)$$

We find that the probability of being scheduled

$$p(r) = \int_{x=0}^{\infty} \left[S_\alpha(x(R/r)^\alpha)\right]^{K-1} s(x)dx \quad (62)$$

and that the conditional bandwidth distribution for a user located at distance r (and the $K-1$ users random located) is given by the results in section II-D with $f \to Nf$ and $S(x) \to S_c(x;r)$ with:

$$S_c(x;r) = \frac{1}{p(r)} \int_{y=0}^{x} \left[S_\alpha(y(R/r)^\alpha)\right]^{K-1} s(y)dx \quad (63)$$

It turns out that extensive simplifications occur for the case where all the users are randomly located in the cell and we find that the distribution of the cell capacity is given by the results in section II-E with $f \to Nf$ and further the α-averaging is given by taking $S_\alpha(x) \to S_\alpha(x)^K$ i.e. is simply the K'th power of the α-averaging of $S(x)$.

C. Combining real-time and non real time traffic over LTE

We are now in the position to combine the analysis in sections III.A and III.B to obtain complete description of the resource usage in a LTE cell. The combined modeling is based on the following assumptions:

- There are M CBR sources applying one of the allocation options described in section III.A.
- There are totally K active (greedy) data sources which always have traffic to send, i.e. we consider the cell in saturated conditions.
- The number of available RBs is taken to be N.

Since the CBR sources have "absolute" priority over the data sources, they will always get the number of RBs they need and hence the leftover RBs will be available for the Non-GBR data sources. By conditioning on the RB usage of the GBR sources we may apply all the results derived in section III.B with available RBs taken to be the leftover RBs not used by the CBR sources. Then we may find the average usage of RBs for the CBR traffic as done in section III.A.

We consider first the case where the location of the sources is given:

- CBR sources are located at distances s_j from the antenna with bit-rate requirements b_j^{CBR}; $j = 1, ..., M$.
- The greedy data sources are located at distance r_i ($i = 1, ..., K$).

With these assumptions the mean cell throughput is given as:

$$\begin{aligned} B_{cell} = \\ &= f\left(N - \sum_{j=1}^{M} \beta(s_j, b_j^{CBR})\right) \sum_{i=1}^{K} \sum_{j=1}^{15} c_j \int_{x=h(r_i,\lambda)g_j}^{h(r_i,\lambda)g_{j+1}} F_i(x,\mathbf{r})s(x)dx + \\ &+ \sum_{j=1}^{M} b_j^{CBR}, \end{aligned} \quad (64)$$

for the discrete bandwidth case and

$$B_{cell} = = d\left(N - \sum_{j=1}^{M}\beta(s_j, b_j^{CBR})\right)\sum_{i=1}^{K}\left(V_i(x,\mathbf{r}) + T\int_{x=g(r_i,\lambda)(e^T-1)}^{\infty} F_i(x,\mathbf{r})s(x)dx\right) + \sum_{j=1}^{M} b_j^{CBR}, \quad (65)$$

for the continuous bandwidth case; where $V_i(x,\mathbf{r}) = \int_{x=0}^{g(r_i,\lambda)(e^T-1)} \ln(1 + x/g(r_i,\lambda))F_i(x,\mathbf{r})s(x)dx$, $\beta(r, b^{CBR})$ is given by by (32) and further $F_i(x,\mathbf{r})$ is defined by (46). For circular cells and power law attenuation on the form $h(r,\lambda) = h(\lambda)r^{\alpha}$ and randomly placed sources the corresponding cell throughput is found to:

$$B_{cell} = = f\left(N - \beta(R, \sum_{j=1}^{M} b_j^{CBR})\right)\frac{2}{R^2}\sum_{j=1}^{15} c_j V_j(x,r) + \sum_{j=1}^{M} b_j^{CBR} \quad (66)$$

where

$$V_j(x,r) = \int_{r=0}^{R} r\left\{\int_{x=h(r,\lambda)g_j}^{h(r,\lambda)g_{j+1}} K\left[\widehat{S}(M(x,r))\right]^{K-1} s(x)dx\right\}dr$$

for the discrete bandwidth case and

$$B_{cell} = = d(N - \beta(R, \sum_{j=1}^{M} b_j^{CBR}))\frac{2}{R^2}\int_{r=0}^{R} r\left\{V(x,r) + T\int_{x=g(r,\lambda)(e^T-1)}^{\infty} K\left[\widehat{S}(M(x,r))\right]^{K-1} s(x)dx\right\}dr + \sum_{j=1}^{M} b_j^{CBR} \quad (67)$$

for the continuous bandwidth case; where $V(x,r) = \int_{x=0}^{g(r,\lambda)(e^T-1)} \ln(1 + x/g(r,\lambda))K\left[\widehat{S}(M(x,r))\right]^{K-1} s(x)dx$, $\beta = \beta(r, b^{CBR})$ is given by (32) and further $\widehat{S}(M(x,r))$ is defined by (53). Observe that the CBR traffic only will affect the cell throughput by the sum $\sum_{j=1}^{M} b_j^{CBR}$ of the rates and not the actual number of CBR sources.

IV. Discussion of Numerical Examples

In the following we give some numerical example of downlink performance of LTE. Before describing the results we first rephrase some of the main assumptions:

- The fading model includes lognormal shadowing (slow fading) and Rayleigh fast fading.
- The noise interference is assumed to be constant over the cell area.

TABLE II
INPUT PARAMETERS FOR THE NUMERICAL CALCULATIONS

Parameters	Numerical values
Bandwidth per Resource Block	180 kHz=12x 15 kHz
Total Numbers of Resource Blocks (RB)	100 RBs for 2Ghz
Distance-dependent path loss. (The actual model is found in [4].)	$L = C + 37.6\log_{10}(r)$, r in kilometers and C=128.1 dB for 2GHz,
Lognormal Shadowing with standard deviation	8 dB (in moust of the cases)
Rayleigh fast fading	
Noise power at the receiver	-101 dBm
Total send power	46.0 dBm=(40W)
Radio signaling overhead	3/14

- The cell shape is circular.

Basically, there are three different cases we would like to investigate. First and foremost is of course the actual efficiency of the LTE radio interface. We choose the bitrate obtainable for the smallest unit available for users, namely a Resource Block (RB). Since different implementation may chose different bandwidth configurations the performance based on RBs will give a good indication of the overall capacity/throughput for the LTE radio interface. Secondly, we know that the scheduling also will affect the overall throughput for a LTE cell. Based on the modeling we are able to investigate the performance of the three basic scheduling algorithms: Round Robin (RR), Proportional Fair (PF) and Max-SINR. All these three algorithms have their weaknesses and strengths, like Max-SINR that try to maximize the throughput but at the cost of fairness among users. Thirdly, we would also investigate the effect on overall performance by introducing GBR traffic in LTE. Normally, GBR traffic will higher priority than Non-GBR or "best effort" traffic and to guarantee a particular rate the number of radio resources required may vary depending on the radio conditions. For users with bad radio conditions i.e. located at cell edge the resource usage to maintain a fixed guaranteed rate may be quite high so an investigation of the cell performance with both GBR and Non-GBR will be important.

A. LTE spectrum efficiency

First, we consider bitrate that is possible to obtainable for the basic resource unit in LTE namely a RB. In the examples we have considered sending frequency of 2 GHz. The aim is to predict the bandwidth efficiency, i.e. the obtainable bitrate per RB. The rest of the input parameters are given in Table 2.

The mean obtainable bitrate per RB is depicted in Figure 3. With our assumptions the maximum bitrate is just below 0.8 Mbit/s for excellent radio conditions. The mean bit-rate is as expected a decreasing function of the cell size both for a randomly placed user and for a user at the cell edge. The mean bitrate have decreased to 0.1 Mbit/s per RB for cell sizes of approximately 2 km for shadowing std. equals 8 dB and when users are random located. The corresponding bit-rate for users at the cell edge is proximately 0.04 Mbit/s.

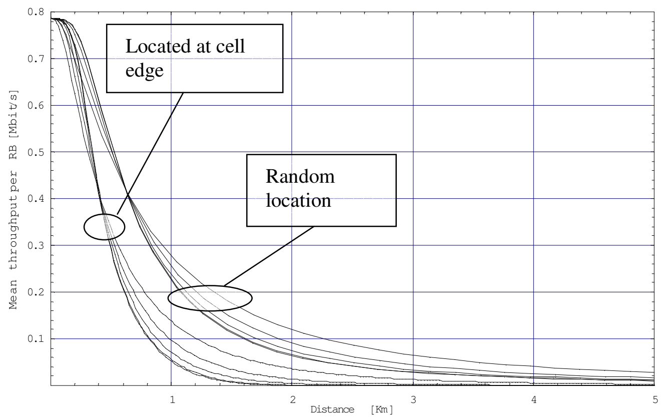

Fig. 3. Mean throughput right and std. left per RB for a user random located, and fixed located as function of cell radius with 2 GHz sending frequency and Suzuki distributed fading with std. of fading σ=0dB, 2dB, 5dB, 8dB, 12dB from below.

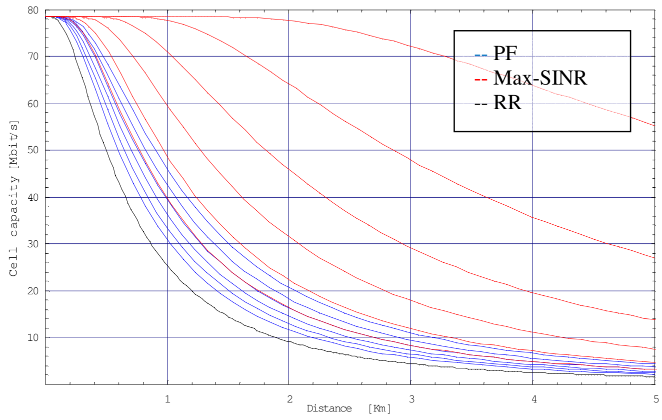

Fig. 4. Multiuser gain as function of cell radius for Max-SINR (red), PF (blue) and RR (black) scheduling, 2GMHz frequency with 100 RB and with Suzuki distributed fading with std. $\sigma = 8$ dB. The number of users is 1, 2, 3, 5, 10, 25, 100 from below.

B. Cell capacity and scheduling

Below we examine the downlink performance in an LTE cell with the input parameters given by Table 2, however, with the following additional input parameters:

- Type of scheduling algorithm i.e. RR, PF or Max-SINR,
- number of RB available i.e. 100,
- number of active (greedy) users.

In Figure 4 the mean downlink cell capacity is depicted as function of the cell radius for RR, PF and the Max SINR scheduling algorithms. As expected the Round Robin algorithm gives the lowest cell throughput while the Max SINR algorithm give the highest throughput. For the latter the multiuser gain is huge and may be explained from the fact that when the number of users increases those who by chance are located near the sender antenna will with high probability obtain the best radio condition and will therefore be scheduled with high bit-rate. For users located at the cell edge the situation is opposite and those users will normally have low SINR and surely obtain very little of the shared capacity. This explains why the Max SINR will increase the throughput but is highly un-fair. For the PF only the relative size of the SINR is important and in this case each user has equal probability of sending in each TTI. The multiuser gain for this algorithm is much lower than for the Max SINR algorithm but is not negligible. When it comes to actual cell downlink throughput the expected values lays in the range 26-48 Mbit/s for cells with radius of 1 km while if the radius is increased to 2 km the cell throughput is reduced to approximately 10-20 Mbit/s.

As seen from Figure 4 the Max-SINR algorithm will over perform the PF algorithm when it comes to cell throughput. But if we consider fairness among users the picture is complete different. When considering the performance of users located at cell edge the Max-SINR algorithm actually performs very badly. While PF give equal probability of transmitting in a TTI for all active users the Max-SINR strongly discriminate the user close to cell edge. As seen from Table 3 below; if there are totally 10 active users in a cell the PF fairly give each user 10% chance of accessing radio resource while the Max-SINR only give 1.2% chance of accessing the radio resources if a user is located at cell edge. As the number of user increases this unfairness increases even more.

TABLE III
PROBABILITY THAT A USER IS SCHEDULED AS FUNCTION OF NUMBERS OF USERS AND LOCATION FOR PF AND MAX-SINR SCHEDULING ALGORITHMS, SUZUKI DISTRIBUTED FADING WITH STD. OF 8DB.

Number of users	PF	MAX-SINR			
		r/R=1	r/R=0.5	r/R=0.25	r/R=0.1
2	0.50	0.308708	0.594756	0.82579	0.96119
3	0.33	0.147869	0.414839	0.71126	0.92784
5	0.20	0.055113	0.245871	0.56102	0.87130
10	0.10	0.012690	0.104912	0.36531	0.76418
25	0.04	0.001356	0.025222	0.16326	0.56989
100	0.01	0.000019	0.001293	0.02453	0.24325

Table 3 demonstrates one of the unfortunate properties of the MAX-SINR scheduling algorithm. While the PF algorithm distribute the capacity among the users with equal probability the MAX-SINR algorithm is far more unfair when it comes to the distribution of the available radio resources. For instance, the users located at the cell edge e.g. r/R=1 will suffer from extremely poor performance if the numbers of users is higher than 10. The Max-SINR algorithm will also be unfair for small cell sizes where users actually may have so high signal quality that most of them may use coding with high data rate i.e. 64 QAM with high rare and there should be no need for scheduling according highest SINR to obtain high throughput.

C. Use of GBR in LTE

It is likely the LTE in the future will carry both real time type traffic like VoIP and elastic data traffic. This is possible by introducing GBR bearers where users are guaranteed the possibility to send at their defined GBR rate. The GBR traffic will have priority over the Non-GBR traffic such that the RBs scheduled for GBR bearers will normally not be accessible for other type of traffic. However, the resource usage over the radio interface in LTE will strongly depends on the radio conditions. This means that the amount of radio resources a user occupies (to obtain a certain bit rate) will vary according to the local radio conditions and a user at the cell edge must seize a larger number of resource blocks (RBs) to maintain a constant rate (GBR bearer) than a user located near the antenna with good radio signals.

An interesting example is to see the effect of multiplexing traffic with both greedy and GBR users and observe the effect on the cell throughput. In Figure 5 we consider the cases where 10 greedy users are scheduled by the PF algorithm together with a GBR user with guaranteed rate of 3, 1, 0.3 or 0.1 Mbit/s. We consider the cases where either the GBR user is located at cell edge or have random location throughout the cell.

We observe that thin GBR connections do not have big impact on the cell throughput. From the figures it seems that GBR bearers up to 1 Mbit/s should be manageable without influencing the cell performance very much. But a 3 Mbit/s GBR connection will lower the total throughput by a quite big factor especially if the user is located at cell edge. For instance we observe for both cases that the effective reduction in cell throughput is approximately 20 Mbit/s for a user requiring a 3 Mbit/s GBR connection when located at cell edge. As a consequence we recommend limiting GBR connection to less than 1 Mbit/s.

We therefore recommend using high GBR values with particular caution. The GBR should be limited to a maximum rate to avoid that a particular GBR user consumes a too large part of the radio resources (too many RBs). A good choice of the actual maximum GBR value seems to be around 1 Mbit/s.

V. Conclusions

With the introduction of LTE the capacity in the radio network will increase considerably. This is mainly due to the efficient and sophisticated coding methods developed during the last decade. However, the cost of such efficiency is that the variation due to radio conditions will increase significantly and hence the possible capacity for users in terms of bitrate will vary a lot depending on the current radio conditions.

The two most important factors for the radio conditions are fading and attenuation due to distance. By extensive analytical modeling where both fading and the attenuation due the distance are included we obtain performance models for:

- Spectrum efficiency through the bitrate distribution per RB for customers that are either randomly or located at a particular distance in a cell.
- Cell throughput/capacity and fairness by taking the scheduling into account.
- Specific models for the three basic types of scheduling algorithms; Round Robin, Proportional Fair and Max SINR.
- Cell throughput/capacity for a mix of GBR and Non-GBR (greedy) users.

Numerical examples for LTE downlink show results which are reasonable; in the range 25-50 Mbit/s for 1 km cell radius at 2GHz with 100 RBs. The multiuser gain is large for the Max-SINR algorithm but also the Proportional Fair algorithm gives relative large gain relative to plain Round Robin. The Max-SINR has the weakness that it is highly unfair in its behaviour.

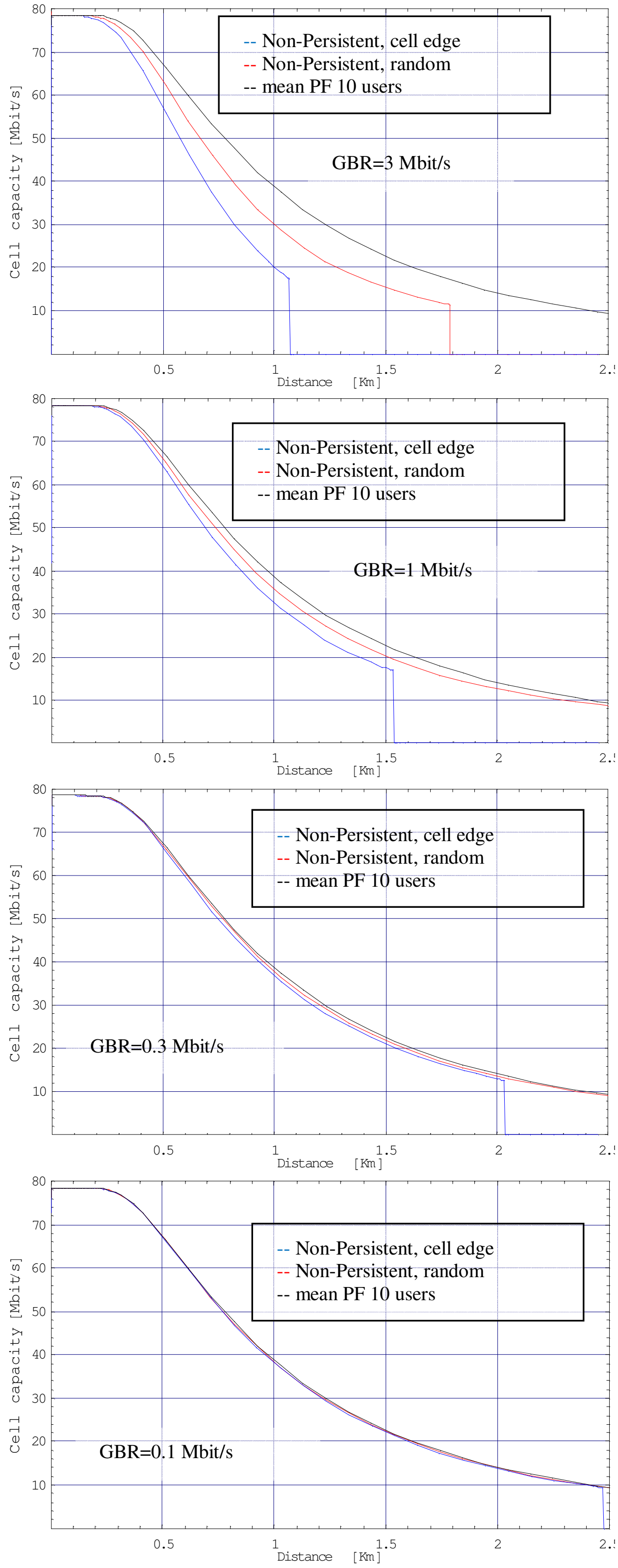

Fig. 5. Mean cell throughput for PF, 10 users and a GBR user of 3.0, 1.0, 0.3, 0.1 Mbit/s using non-persistent scheduling, for 2 GHz and 100 RB and Suzuki distributed fading with std. $\sigma = 8$dB. Red curves corresponds to random location and blue for user located at cell edge.

User at cell edge with poor radio condition will obtain very little data throughput. It turns out that the grade of unfairness increases with the numbers of active users. This unfortunate property is not found for the Proportional Fairs scheduling algorithm.

The usage of GBR with high rates may cause problems in LTE due to the high demand for radio resources if users have low SINR i.e. at cell edge. For non-persistent GBR allocation the allowed guaranteed rate should be limited. It seems that a limit close to 1 Mbit/s will be a good choice.

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