# MAC Layer Outage Probability of Bounded Ad Hoc Networks

Mariam Kaynia, Flavio Fabbri, Geir E. Øien, and Roberto Verdone

*Abstract*—**We consider a bounded square-shaped ad hoc network scenario, within which packet arrivals are distributed randomly in space and time according to a 3-dimensional Poisson Point Process. Each packet is transmitted over a single hop to its destination, located a fixed distance away. Within this context, the outage probability of the ALOHA and CSMA protocols is derived, and we evaluate the impact of edge effects in space on the performance of MAC protocols. Our analytical expressions are verified with Monte Carlo simulations. The behavior of the network is evaluated as the system parameters, such as the node density, the physical size of the network, and the distance between each transmitter and its receiver, vary. Furthermore, the obtained results are compared to those of unbounded networks, showing that edge effects reduce the average outage probability across the network significantly, due to the lower level of interference suffered by boundary nodes.**

## I. Introduction

Wireless ad hoc networks have undergone extensive analysis by numerous researchers throughout the years. Various system models have been applied, and the inherent problems of ad hoc networks, such as interference and the random topology, are addressed from different design perspectives. One popular way to manage the allocation of available radio resources in such an unpredictable context, is through random Medium Access Control (MAC) protocols; they are characterized by a distributed nature, where each node tries to access the shared medium without knowledge of the network topology and of the other nodes' activation status. Among the most common MAC protocols are ALOHA and Carrier Sensing Multiple Access (CSMA). In this paper, we evaluate the MAC layer outage probability (OP) in a wireless ad hoc network where nodes are uniformly and randomly distributed in a bounded square-shaped domain and packet arrivals follow a Poisson process. OP is defined as the probability that packets either do not reach the receiver (due to dropping of the packet at the transmitter after all access attempts are failed) or are received with errors during all their retransmission attempts. The impact on performance due to edge effects caused by the boundedness of the network is evaluated. The MAC layer OP includes the effects of both the path loss and the MAC protocol procedures (backoffs and retransmissions). Also, it is easily related to other performance metrics, such as throughput and transmission capacity [1].

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The network model used in our work resembles the physical model of Gupta and Kumar's interference channel [2], with the difference that we also allow for unslotted transmissions and multiple retransmissions. In [1], Weber *et al.* consider a stochastic SINR-based model similar to that of [2], within which they evaluate the density of transmissions using slotted ALOHA with a given outage constraint. By adding carrier sensing capabilities to ALOHA, many works have addressed the mathematical characterization of CSMA in its standardized evolution [3], [4]. However, in all these works, an infinite plane is assumed, and thus, the impact of edge effects is ignored. Other papers also evaluate the performance of the various MAC protocols [5], [6], [7], [8]. The closest work to ours, is that of [5], where the OP of the ALOHA and CSMA MAC protocols is derived in an unbounded homogeneous ad hoc network with no fading effects.

Assuming an *unbounded* network, as was done in the previously mentioned works, is not always an appropriate model. In many ad hoc networks, such as military battlefields, emergency scenarios, or sensor networks, the communication domain is bounded. Interference, being one of the most challenging issues in the design of ad hoc networks, is closely related to the network topology, the area of the communication domain, and the density of nodes. The physical size and shape of the network, as well as the location of the node of interest, plays an essential role in the amount of interference that this node detects.

Only a few works have evaluated the significance of the boundedness of regions and the impact of the node position. In [9], distance distributions of uniformly and Gaussian distributed nodes in a rectangular area are considered. Other network types and shapes, such as Manhattan networks, hypercubes, and shufflenets are investigated in [10]. In [11], the capacity of networks with a regular structure, using slotted ALOHA, is considered. For linear networks, it is shown that the capacity is almost constant with respect to the network size. In two-dimensional networks, on the other hand, capacity grows in proportion to the square root of the area of the deployment region [11].

The edge effects due to finiteness of the deployment region have been considered in [12]. Extensions of this work can be seen in [13], where the specific case of a square domain is analyzed in detail. However, the two latter works only focus on connectivity, while no indication is given to its implications on the performance at MAC layer. In [14], the impact of edge effects is evaluated in a bounded square network with focus on the MAC layer. However, the MAC protocols considered in [14] are simplified in that no backoffs or retransmissions are allowed. Our work serves as an extension to [14] by allowing for multiple backoffs and retransmissions, something that changes the analysis and final results considerably.

## II. Network Model

Our system model resembles that of [5], with the difference of having a bounded region. Consider a finite 2-D square, denoted as $\mathcal{D}$, of size $L \times L$, where transmitter nodes are deployed according to a homogeneous Poisson Point Process (PPP) with spatial density $\lambda^s$ [nodes/m$^2$]. At each transmitter, data packets of a fixed duration $T$ are formed according to an independent 1-D Poisson process with temporal density $\lambda^t$ [packets/sec/node]. Upon the formation of each packet, it is transmitted with constant power $\rho$ to its intended receiver, which is located a fixed distance $R$ away. Assuming $R$ to be fixed can be justified by viewing this network as a snapshot of a multi-hop wireless network, where $R$ is the inter-relay distance designed by the routing protocol.

The sources of randomness in this ad hoc network are the positions and number of active transmitters at each time instant. The spatial density of *new* packet arrivals during an interval of duration $T$ is $\lambda = \lambda^s \lambda^t T$ [packets/m$^2$]. The expected number of transmitters attempting to access the channel in $\mathcal{D}$ is $z_D = \lambda L^2(1 + \Delta(M,N))$, where $\Delta(M,N)$ represents the amount of increase in the spatial density of packets due to $M$ backoffs and $N$ retransmissions, depending on the applied protocol; if we are interested in a sub-region $\mathcal{A}$ of size $A < L^2$, the number of packets in the network is $z_A = \lambda A(1 + \Delta(M,N))$. In the ALOHA protocol, packets are transmitted to their intended destinations regardless of the channel condition. With CSMA, channel sensing is performed at the *beginning* of each packet and a decision is made on whether or not the transmission should be initiated; if the SINR is below the required threshold $\beta_b$,[1] the packet is backed off a random time. With both ALOHA and CSMA, the packet is considered received with errors if the SINR is below the required threshold $\beta$ at any instant during the packet transmission interval, $T$.

In our network model, we assume high mobility of nodes, meaning that different independent sets of packets are active between times $t_0$ and $t_0 + T$. Furthermore, the waiting time from one transmission attempt to the next is set to be larger than $T$. As a consequence of these model characteristics, we may assume that there are no spatial and temporal correlations between the various transmission attempts of a packet.

In order to address the problem of edge effects, we consider only those packets that fall inside the square region as part of the study. The origin of our coordinates system is placed in the center of the square. For the sake of the analysis carried out in the next sections, we propose a tessellation of such a domain featuring eight subregions, as shown in Fig. 1. This will be described in more details in Section III.

For the channel model, we consider only deterministic path loss attenuation effects (with path loss exponent $\alpha > 2$). That is, additional fading effects are ignored. Each receiver potentially sees interference from all transmitters, and these independent interference powers are added to the channel noise $\eta$, resulting in

[1]The subscript $b$ refers to this threshold being used for the *b*ackoff decision.

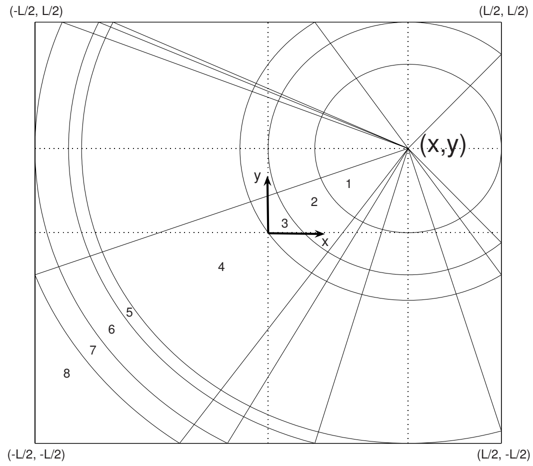


Fig. 1. Partitioning of the finite area to derive the OP in bounded regions.

$$\text{SINR} = \frac{\rho\, c\, R^{-\alpha}}{\eta + \sum_i \rho\, c\, r_i^{-\alpha}}, \tag{1}$$

where $r_i > 0$ is the distance between the node under observation (this could be either the transmitter or receiver of the packet we are considering) and the $i$-th interfering transmitter, and the sum is over all active interferers on the plane at a given time instant; $c$ is the channel gain at a unit distance away from the transmitter, and in the rest of the paper we set it to 1 without loss of generality. If the received SINR falls below the threshold $\beta$ at any time during a packet transmission, that packet is said to be received *erroneously* with probability

$$P_{error} = \Pr(\text{SINR} < \beta). \tag{2}$$

Similary, the backoff probability is $P_b = \Pr(\text{SINR} < \beta_b)$.

The performance metric applied in this work is the *OP*, derived as different combinations of the error probability depending on $N$ and $M$. In the general sense, the OP of ALOHA and CSMA is defined mathematically as

$$\begin{aligned} P_{out}(\text{ALOHA}) &= \Pr[\text{SINR} < \beta \text{ during } [0,T)\ N+1 \text{ times}], \\ P_{out}(\text{CSMA}) &= \Pr\big[\text{SINR} < \beta_b \text{ at } t=0\ M \text{ times} \\ &\qquad \cup\ \text{SINR} < \beta \text{ during } [0,T)\ N+1 \text{ times}\big]. \end{aligned} \tag{3}$$

## III. Outage Probability Analysis

The evaluation of the OP consists of solving Eq. (3) in the specific case of the square domain $\mathcal{D}$. However, the consideration of all interfering contributions in the denominator of Eq. (2) turns out to be impractical due to many sources of randomness (i.e., space, time, and number of interferers). For this reason we focus on the probability of having a single *closest* interferer, whose received interference power alone is strong enough to result in outage for the packet of interest. Considering only the occurrence of a single interferer causing outage for an ongoing transmission, provides a *lower bound* to the OP, which has been shown to be fairly tight around the actual performance [1].

TABLE I
BOUNDARY VALUES FOR $r$ IN THE EIGHT REGIONS

| Region | Range: $r_{1,i}(x,y) \le r \le r_{2,i}(x,y)$ |
|---|---|
| 1 | $0 \le r \le \frac{L}{2}-x$ |
| 2 | $\frac{L}{2}-x \le r \le \frac{L}{2}-y$ |
| 3 | $\frac{L}{2}-y \le r \le \sqrt{(\frac{L}{2}-x)^2+(\frac{L}{2}-y)^2}$ |
| 4 | $\sqrt{(\frac{L}{2}-x)^2+(\frac{L}{2}-y)^2} \le r \le \frac{L}{2}+y$ |
| 5 | $\frac{L}{2}+y \le r \le \sqrt{(\frac{L}{2}-x)^2+(\frac{L}{2}+y)^2}$ |
| 6 | $\sqrt{(\frac{L}{2}-x)^2+(\frac{L}{2}+y)^2} \le r \le \frac{L}{2}+x$ |
| 7 | $\frac{L}{2}+x \le r \le \sqrt{(\frac{L}{2}+x)^2+(\frac{L}{2}-y)^2}$ |
| 8 | $\sqrt{(\frac{L}{2}+x)^2+(\frac{L}{2}-y)^2} \le r \le \sqrt{(\frac{L}{2}+x)^2+(\frac{L}{2}+y)^2}$ |

TABLE II
BOUNDARY VALUES FOR THE ANGLE $\theta$ IN THE EIGHT SUBREGIONS

| Region | $\theta_i(r,x,y)$ |
|---|---|
| 1 | $\pi$ |
| 2 | $\frac{\pi}{2}+\arcsin\frac{\frac{L}{2}-x}{r}$ |
| 3 | $\frac{\pi}{2}+\arcsin\frac{\frac{L}{2}-x}{r}-\arccos\frac{\frac{L}{2}-y}{r}$ |
| 4 | $\frac{\pi}{2}+\frac{1}{2}(\arcsin\frac{\frac{L}{2}-x}{r}-\arccos\frac{\frac{L}{2}-y}{r})$ |
| 5 | $\frac{\pi}{2}-\arccos\frac{\frac{L}{2}+y}{r}+\frac{1}{2}(\arcsin\frac{\frac{L}{2}-x}{r}-\arccos\frac{\frac{L}{2}-y}{r})$ |
| 6 | $\frac{\pi}{2}-\frac{1}{2}(\arccos\frac{\frac{L}{2}+y}{r}+\arccos\frac{\frac{L}{2}-y}{r})$ |
| 7 | $\frac{1}{2}(\arcsin\frac{\frac{L}{2}-y}{r}+\arcsin\frac{\frac{L}{2}+y}{r})-\arccos\frac{\frac{L}{2}+x}{r}$ |
| 8 | $\frac{1}{2}(\arcsin\frac{\frac{L}{2}+y}{r}-\arccos\frac{\frac{L}{2}+x}{r})$ |

In order to consider the closest interferer as the determinant for OP, we first recall the notion of guard zones [15]. Consider the active transmission of the transmitter-receiver pair $TX_0$-$RX_0$. The guard zone $B(RX_0, s)$ is a circle centered on $RX_0$ with radius $s$, defined such that if a single interferer, $TX_i$, is located at a distance less than $s$ away from $RX_0$, the packet of $TX_0$ will be received erroneously. The radius $s$ is derived from Eq. (1) as

$$s = \left(\frac{R^{-\alpha}}{\beta} - \frac{\eta}{\rho}\right)^{-\frac{1}{\alpha}}. \quad (4)$$

Based on this expression, $\beta_b$ corresponds to $s_b$. However, in the following, we will assume that $\beta_b = \beta$. For more results on the OP when $\beta_b \neq \beta$ (but in an infinite network), we refer the reader to [5]. With the guard zone concept, our problem can be reduced to a distance problem. That is, we are now concerned with finding the areas of intersection between the circles $B(RX_0, s)$, $B(TX_0, s)$, and the square domain $\mathcal{D}$ for every $(x, y)$-coordinate of the network.

In order to account for edge effects, we must analyze the OP at every given $(x, y)$-position. For this, we propose a tessellation of our square-shaped plane featuring eight subregions, as shown in Fig. 1. The advantage of this is that it allows us to switch to polar coordinates easily, owing to the decomposition of the surface into sectors of annuli. Given whatever position $(x, y)$ inside the square, only a fraction of the surface of a circle of radius $r$ centered in $(x, y)$ lies in $\mathcal{D}$. This fraction is $F = \frac{\theta_i(r,x,y)}{\pi}$, where $\theta_i(r, x, y)$ is dependent on the size of $r$, which ranges for each subregion $i$ of the 8 sectors of $\mathcal{D}$ from $r_{1,i}(x, y)$ to $r_{2,i}(x, y)$. These are defined in Table I, while $\theta_i(r, x, y)$ is given in Table II. These distances and angles are used in the derivations of the OP in the following subsections. The entries of Tables I and II were derived in [13].

The following subsections provide theorems that give the OP for ALOHA and CSMA.

## A. Slotted ALOHA

ALOHA is the simplest form of MAC protocols. It was proposed in 1970 by N. Abramson [16] and was the start of a series of MAC protocol designs. In ALOHA, packets are transmitted to their destinations regardless of the channel conditions and the OP. In slotted ALOHA, the time domain is divided in slots, and each packet can only be transmitted at the start of the first slot after it has been formed. The disadvantage of slotted protocols is the need for synchronization; not only does this introduce delays in the system, but it is also costly and challenging to maintain in a network that is in constant change. For networks that have no synchronization capability, unslotted protocols are employed. Each packet is given a maximum of $N$ retransmission attempts before the link is counted to be in outage.

The OP of slotted ALOHA is given by the following theorem.

*Theorem 1:* The OP of slotted ALOHA can be lower bounded by $P_{out,lb}(\text{Slotted ALOHA}) = \overline{P}_{rt,sl}^{N+1}$, with $\overline{P}_{rt,sl}$ as the solution to

$$\overline{P}_{rt,sl} = \mathbb{E}_{x,y}\left[1 - e^{-\lambda \frac{1-\overline{P}_{rt,sl}^{N+1}}{1-\overline{P}_{rt,sl}} \sum_{i=1}^{8} \int_{r_{1,i}(x,y)}^{r_{2,i}(x,y)} 2\,\theta_i(r,x,y)\, r\, u(s-r)\, dr}\right], \quad (5)$$

where $u(\cdot)$ is the step function, and $r_{1,i}(x, y)$, $r_{2,i}(x, y)$, and $\theta_i(r, x, y)$ are given in Tables I and II, respectively.

*Proof:* The proof of Theorem 1 is given in Appendix A ($\beta$ is given implicitly in $s$ through Eq. (4)). ■

## B. Unslotted ALOHA

In unslotted ALOHA, packets are transmitted to their destinations immediately upon their formation. This gives rise to the the problem of partial overlap of packets. Compared to slotted ALOHA, the OP of unslotted ALOHA is increased as we now need to consider 2 time slots in order to account for the partial overlap of packets. The outage probability is given by the following theorem.

*Theorem 2:* The OP of unslotted ALOHA can be lower bounded by $P_{out,lb}(\text{Unslotted ALOHA}) = \overline{P}_{rt,u}^{N+1}$, where $\overline{P}_{rt,u}$ is the solution to

$$\overline{P}_{rt,u} = \mathbb{E}_{x,y}\left[1 - e^{-2\lambda \frac{1-\overline{P}_{rt,u}^{N+1}}{1-\overline{P}_{rt,u}} \sum_{i=1}^{8} \int_{r_{1,i}(x,y)}^{r_{2,i}(x,y)} 2\,\theta_i(r,x,y)\, r\, u(s-r)\, dr}\right], \quad (6)$$

where $u(\cdot)$ is the step function, and $r_{1,i}(x, y)$, $r_{2,i}(x, y)$, and $\theta_i(r, x, y)$ are given in Tables I and II, respectively.

*Proof:* The proof of Theorem 2 is given in Appendix A. ■

### C. CSMA With Transmitter Sensing

In order to improve the performance of ALOHA, the CSMA protocol was proposed by Kleinrock and Tobagi in 1975 [17]. In CSMA, the channel is sensed prior to transmission, and if the measured or estimated (depending on which node performs the sensing) Signal-to-Interference-plus-Noise Ratio (SINR) is below a given threshold, the packet is backed off a random time, before a new channel sensing is performed. Once the transmission is initiated, but the received packet contains errors, it is retransmitted. Each packet is given a maximum of $M$ backoffs and $N$ retransmission attempts before the packet is dropped and the link is counted to be in outage.

CSMA with transmitter sensing, denoted $\text{CSMA}_{\text{TX}}$, is the conventional CSMA protocol where the *transmitter* performs the channel sensing and based on the SINR that it estimates at its receiver, makes the backoff decision.

Due to the sensing at transmitters prior to each packet transmission, the OP becomes a combination of the backoff probability and the probability that outage occurs during an active transmission. This probability is given by the following theorem.

*Theorem 3:* The OP of $\text{CSMA}_{\text{TX}}$ is given by

$$\overline{P}_{out}(\text{CSMA}_{\text{TX}}) = \overline{P}_b^M + (1-\overline{P}_b^M)\,\overline{P}_{rt1}\,\overline{P}_{rt}^N, \tag{7}$$

where:

- $\overline{P}_b = \mathbb{E}_{x,y}\left[P_b(x,y)\right]$ is the average backoff probability, approximated by the solution to

$$\overline{P}_b \;=\; \mathbb{E}_{x,y}\left[1 - e^{-\lambda_{active}^{TX}\sum_{i=1}^{8}\int_{r_{1,i}(x,y)}^{r_{2,i}(x,y)} 2\theta_i(r,x,y)\, r\, u(s-r) dr}\right], \tag{8}$$

  with $\lambda_{active}^{TX}$ given as

$$\lambda_{active}^{TX} = \lambda\left(1-\overline{P}_b^M + (1-\overline{P}_b^M)\overline{P}_{rt1}\frac{1-\overline{P}_{rt}^N}{1-\overline{P}_{rt}}\right). \tag{9}$$

- $\overline{P}_{rt} = \mathbb{E}_{x,y}\big[P_b(x,y) + (1-P_b(x,y))P_{during}^{TX}(x,y)\big]$ is the average probability that a packet is received in error during a retransmission attempt, with

$$P_{during}^{TX}(x,y) \approx \sum_{i=1}^{8}\int_{r_{1,i}(x,y)^2}^{r_{2,i}(x,y)^2} \theta_i(r,x,y)\, e^{-\lambda_{csma}^{TX}\,\theta_i(r,x,y)\, r^2}\times \lambda_{csma}^{TX}\left[1-\tfrac{1}{\pi}\cos^{-1}\!\left(\tfrac{r^2+R^2-s^2}{2Rr}\right)\right]\text{rect}\left(\tfrac{r-s+R/2}{R}\right)dr^2, \tag{10}$$

  where $\text{rect}\left(\frac{t-a}{b}\right) = u\left(t-a+\frac{b}{2}\right) - u\left(t-a-\frac{b}{2}\right)$, while $r_{1,i}(x,y)$, $r_{2,i}(x,y)$, and $\theta_i(r,x,y)$ are given in Tables I and II, respectively.
- $\overline{P}_{rt1} = \mathbb{E}_{x,y}\left[\overline{P}_{rx|active} + \left(1-\overline{P}_{rx|active}\right)P_{during}^{TX}(x,y)\right]$ is the average probability that the packet is received in outage at its first transmission attempt, with

$$\overline{P}_{rx|active} \approx \overline{P}_b\left[1-\tfrac{1}{\pi s^2}\left(2s^2\cos^{-1}\!\left(\tfrac{R}{2s}\right) - Rs\sqrt{1-\tfrac{R^2}{4s^2}}\right)\right]. \tag{11}$$

- The density of packets attempting to access the channel is given by

$$\lambda_{csma}^{TX} = \lambda\left[\frac{1-\overline{P}_b^M}{1-\overline{P}_b} + (1-\overline{P}_b^M)\,\overline{P}_{rt1}\,\frac{1-\overline{P}_{rt}^N}{1-\overline{P}_{rt}}\right]. \tag{12}$$

  *Proof:* The proof of Theorem 3 is given in Appendix B. ■

### D. CSMA With Receiver Sensing

In order to ameliorate the performance of CSMA, a modified version, $\text{CSMA}_{\text{RX}}$, was introduced in [5], where the *receiver* performs channel sensing and makes the backoff decision. This nullifies the inherent exposed node problem of $\text{CSMA}_{\text{TX}}$, but adds to the hidden node problem. The exposed node problem occurs when a transmitter decides to back off in cases when its receiver would have received the packet successfully. The communication between the receiver and its transmitter requires a control channel, which is presumed to be orthogonal to the data channel, such that there are no interference issues between control and data signals.[2]

Using the techniques of Section III-C, we derive the OP of $\text{CSMA}_{\text{RX}}$, as stated in the following theorem.

*Theorem 4:* The average OP of $\text{CSMA}_{\text{RX}}$ is given by

$$\overline{P}_{out}(\text{CSMA}_{\text{RX}}) \;=\; \overline{P}_b^M + (1-\overline{P}_b^M)\,\overline{P}_{rt1}^{RX}\,\overline{P}_{rt}^N, \tag{13}$$

where

- $\overline{P}_b = \mathbb{E}_{x,y}\left[P_b(x,y)\right]$ is the average backoff probability, approximated by the solution to Eq. (8), with $\overline{P}_{rt1} = \overline{P}_{during}$.
- $\overline{P}_{rt} = \mathbb{E}_{x,y}\big[P_b(x,y) + (1-P_b(x,y))\,P_{during}^{RX}(x,y)\big]$ is the approximate average probability that a packet is received in error during a retransmission attempt, with

$$P_{during}^{RX}(x,y) \;\approx\; \sum_{i=1}^{8}\int_{r_{1,i}(x,y)^2}^{r_{2,i}(x,y)^2}\int_{\nu(r)}^{\gamma(r)} \frac{\lambda_{csma}^{RX}}{2\pi} P(\text{active}|r,\phi) \times\theta_i(r,x,y)\, e^{-\lambda_{csma}^{RX}\,\theta_i(r,x,y)\, r^2}\, u(s-d)\, d\phi\, d(r^2), \tag{14}$$

  where $r_{1,i}(x,y)$, $r_{2,i}(x,y)$, and $\theta_i(r,x,y)$ are given in Tables I and II, respectively. $P(\text{active}|r,\phi)$, $\nu(r)$ and $\gamma(r)$ are given as:

$$P(\text{active}|r,\phi) = 1 - \frac{1}{\pi}\cos^{-1}\!\left(\frac{r^2+2R^2-s^2-2Rr\cos\phi}{2R\sqrt{r^2+R^2-2Rr\cos\phi}}\right),$$
$$\nu(r) = \cos^{-1}\!\left(\frac{r^2+2Rs-s^2}{2Rr}\right) \quad\wedge\quad \gamma(r) = 2\pi - \nu(r). \tag{15}$$

- $\overline{P}_{rt1} = \overline{P}_{during}$, and the density of packets attempting to access the channel is given by Eq. (12), with $\overline{P}_b$ and $\overline{P}_{rt}$ as given above.

  *Proof:* The proof of Theorem 4 is given in Appendix C. ■

[2] Note that the main difference between the proposed $\text{CSMA}_{\text{RX}}$ protocol and the CSMA/CA protocol used in the IEEE 802.11 and 802.16 standards is that in the latter, *all* nodes hear the request-to-send (RTS) and clear-to-send (CTS) signals, whereas in $\text{CSMA}_{\text{RX}}$, we assume that the communication of control signals is between a receiver and its own transmitter only.

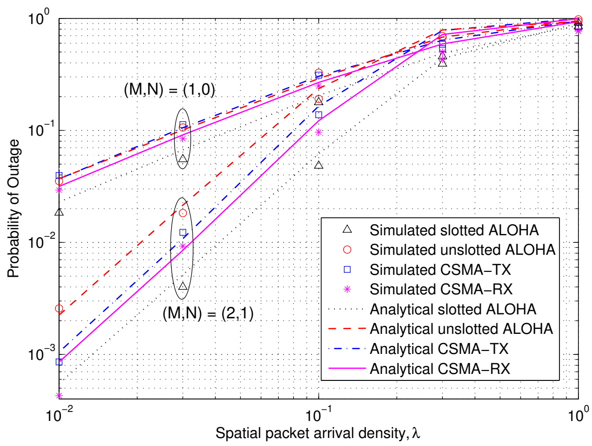


Fig. 2. Analytical and simulated OP of ALOHA and CSMA in a bounded network, as a function of $\lambda$.

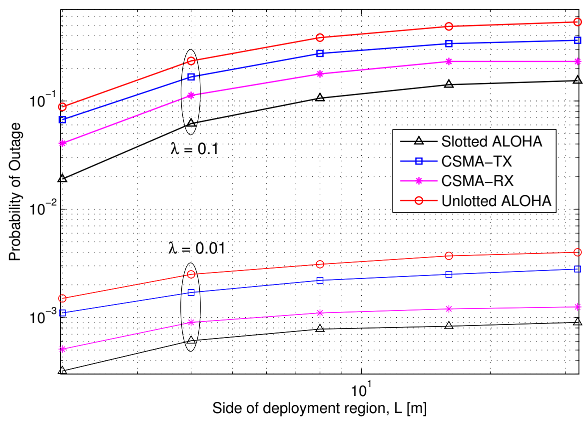


Fig. 3. OP of ALOHA and CSMA in a bounded network as a function of the size of the deployment region, $L$.

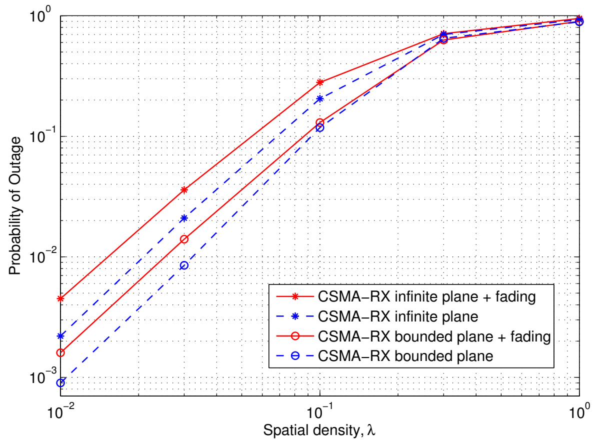


Fig. 4. OP of $\text{CSMA}_{\text{RX}}$ for bounded and unbounded networks, both in the absence and presence of fading, for $(M, N) = (2, 1)$.

## IV. Numerical Results

Monte Carlo simulations are generated to confirm our derived expressions in Section III. The simulation model is as described in Section II. Unless stated otherwise, we use the following (normalized) parameter values: fixed transmitter-receiver distance $R = 1$ m, transmission power $\rho = 1$ mW, path-loss exponent $\alpha = 4$, SINR threshold $\beta_b = \beta = 1$, number of backoffs $M = 2$, and number of retransmissions $N = 1$.

Fig. 2 shows the OP of the various MAC protocols as a function of the transmission density $\lambda$. The simulations are seen to follow the analytical results tightly, thus validating our derivations. As the density increases, so does the OP, until it reaches a saturation point above which the OP is approximately 1. This point of saturation is about the same for all protocols and for both $(M, N) = (1, 0)$ and $(M, N) = (2, 1)$; this is because at this point, the interferer density is so high that introducing carrier sensing or multiple backoffs and retransmissions does not provide any visible performance gain.

Slotted ALOHA is seen to yield the lowest OP, due to the avoidance of partial overlap of packets. Among the unslotted protocols, $\text{CSMA}_{\text{RX}}$ performs the best, because by allowing the receiver to make the backoff decision, there is no exposed node problem. The exposed node problem occurs when a node pair decides to back off in situations where its packet would have been received correctly at the receiver. Interestingly, for $(M, N) = (1, 0)$ and low densities, $\text{CSMA}_{\text{TX}}$ appears to perform even worse than unslotted ALOHA, by approximately 10%. This is due to the exposed node problem, which is significantly reduced by allowing for more backoff attempts. When $(M, N) = (2, 1)$, the difference between the protocols increases, and $\text{CSMA}_{\text{TX}}$ outperforms unslotted ALOHA by 30%, while it is outperformed by $\text{CSMA}_{\text{RX}}$ by 50%. As the density increases, so does the backoff probability, and the amount of interference is thus lower in the case of CSMA than ALOHA. Hence, for higher densities, the benefit of the carrier sensing capability of CSMA becomes more evident.

Having validated our model and analytical results, we now evaluate the system performance as the size of the deployment region is changed. In Fig. 3, the analytical expression for OP is plotted as a function of the length of the side of our square region, $L$, for fixed densities of $\lambda = 0.01$ and $0.1$ [packets/m$^2$]. At the high density, up to 85% of the OP is reduced due to edge effects when the side of the deployment region is reduced to the same order of the guard zone radius, $s$. Our simulations show that the OP of a node located in the corner of the domain is about 50% lower than the one located in the center.

In Fig. 4, we compare the OP of a bounded region (based on the analytical expressions derived in Section III) to that of an unbounded one, both for a non-fading network and a fading one (which in not analyzed in this manuscript; results are taken for comparison purposes from [18]). We only consider $\text{CSMA}_{\text{RX}}$, as the other protocols behave in the same manner. In the absence of fading, the bounded network with side length $L = 3.3$ yields up to 60% lower OP than the unbounded network. The degradation due to fading is about 47% in unbounded networks and about 38% in bounded networks. The reason for this difference is that there is a greater number

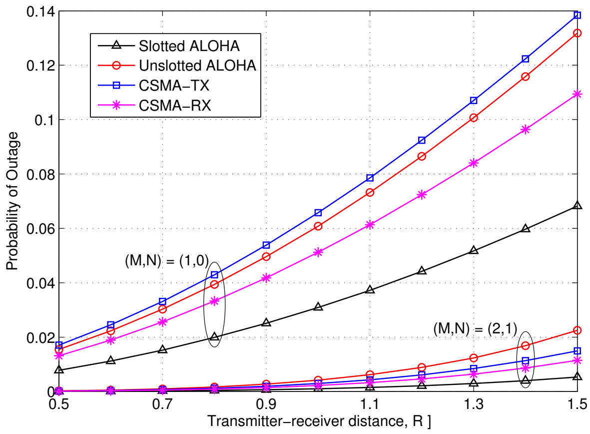


Fig. 5. Dependence of the OP on the transmitter-receiver distance $R$ for $\lambda = 0.01$.

of interferers in the unbounded network. Consequently, when each interferer causes a greater destruction due to fading, the aggregate destruction becomes more significant as the number of interferers increases. Hence, the impact of fading is more severe in infinite networks compared to bounded ones.

Finally, in Fig. 5, we investigate the behavior of the OP as the transmitter-receiver distance, $R$, varies. For higher values of $R$, the OP increases approximately linearly. Moreoever, we note that a change in $R$ has a greater impact on the outage probability for smaller values of $M$ and $N$.

## V. Conclusions

We consider a bounded ad hoc network and evaluate the impact of edge effects on the performance of MAC protocols. Our model depicts an ad hoc network where transmitter nodes are distributed in finite space according to a Poisson point process (PPP), with packets arriving in time according to a 1-D Poisson process. The network performance is evaluated in terms of outage probability (OP), which is defined as the probability that the received SINR falls below a given threshold required for correct decoding of packets. The MAC protocols ALOHA and CSMA (in their various incarnations) are applied for communication between links, and approximate expressions are derived for the OP of these protocols.

Our analytical results allow for several number of backoffs and retransmissions, varying transmission densities, different sizes of the deployment region, and any sensing threshold imposed by the system. Monte Carlo simulations are generated to validate our derivations, and the performance of the various MAC protocols are analyzed and compared. When fewer backoffs are allowed compared to the number of retransmissions, slotted ALOHA is shown to yield the best performance in our bounded network, as in infinite networks. However, if the network has no synchronization capabilities, then the most reliable protocol to use is CSMA with receiver sensing, $\text{CSMA}_{\text{RX}}$. This protocol is modified from the conventional CSMA protocol by introducing a simple feedback channel. It is also observed that the OP is much lower in bounded networks compared to infinite ones. Specifically, edge effects reduce the OP by up to 85%.

For future work, we wish to extend our model to a multi-hop network, and investigate the impact of edge effects on the system performance. The number of and distance between hops will then be optimized in order to minimize the OP. It is also worth noting that when performing carrier sensing, we assume in this work that the sensing node measures the SINR. This can be assumed provided that the node has knowledge of the level of desired received power due to signaling over an orthogonal channel. This assumption will be removed in future works.

## Appendix

### A. *Proof of Theorems 1 and 2*

Observing the packet transmission of a randomly chosen transmitter-receiver pair $\text{TX}_0$-$\text{RX}_0$ (where $\text{RX}_0$ is in $(x, y)$), starting at time 0, we know that with slotted ALOHA, all packet arrivals during the time period $[-T, 0)$ are interferers. Moreover, with the concept of guard zones to derive a lower bound to the outage probability, all packet arrivals during $[-T, 0)$ inside $\text{RX}_0$'s guard zone, $B(\text{RX}_0, s)$, can cause outage for the packet reception at $\text{RX}_0$. Hence, we need to evaluate the average number of packet arrivals during $[-T, 0)$ inside the area $\mathcal{A} = \mathcal{D} \cap B(\text{RX}_0, s)$, denoting it as $z_A$. The outage probability, conditioned on $s$ and $(x, y)$, is given by $1 - \exp\{-z_A\}$. Since nodes are uniformly distributed in space, we may then take the average of Eq. (5); therefore, $\overline{P}_{rt,sl} = \mathbb{E}_{x,y}[1 - \exp\{-z_A\}]$.

As mentioned in Section II, $z_A = \lambda A(1 + \Delta(M, N))$ where $A$ is the size of $\mathcal{A}$, which in this case (owing to the tessellation into eight subregions of the square domain) is equal to $\sum_{i=1}^{8} \int_{r_{1,i}(x,y)}^{r_{2,i}(x,y)} 2\,\theta_i(r, x, y)\, r\, u(s - r)\, dr$.

Next, we evaluate the density of interferers, $\lambda_{slotted} = \lambda(1 + \Delta(M, N))$. We know that the number of interferers inside $\mathcal{A}$, when $\text{RX}_0$ is in $(x, y)$, is Poisson distributed with density $\lambda_{slotted}$. Allowing for retransmissions is equivalent to increasing the number of packets that attempt to access the channel. Since the waiting times $t_{wait}$ are random and uncorrelated (due to the high mobility of nodes, and by assuming that $t_{wait} > T$), there is no correlation between the amount of interference detected in each retransmission attempt. Denoting the average (over $x$ and $y$), the probability of a packet being retransmitted in slotted ALOHA as $\overline{P}_{rt,sl} = \mathbb{E}[P_{rt,sl}(x, y)]$, with $P_{rt,sl}(x, y)$ being the corresponding probability conditioned to $(x, y)$, the density of active packets in the channel is

$$\begin{aligned}\lambda_{slotted} &= \lambda\left(1 + \overline{P}_{rt,sl} + \overline{P}_{rt,sl}^2 + \ldots + \overline{P}_{rt,sl}^N\right)\\ &= \lambda\frac{1 - \overline{P}_{rt,sl}^{N+1}}{1 - \overline{P}_{rt,sl}}. \end{aligned} \tag{16}$$

Finally, a packet is in outage if it is received erroneously in all $N + 1$ transmission attempts, resulting in Theorem 1.

The proof for Theorem 2 is similar, with the only difference that in the unslotted case, we have to consider all packet arrivals during $[-T, T)$. That is, the continuity of packet transmissions in time gives rise to partial overlap of packets. Note that the number of packet arrivals (and hence, the amount of interference) in $[-T, 0)$ is independent from that in $[0, T)$, and this is why the only difference between Eqs. (5) and (6) is the factor 2 in the exponent of the $\exp(\cdot)$-expression. The density of interferers in unslotted ALOHA is $\lambda_{unslotted}$, given by Eq. (16), with $\overline{P}_{rt,sl}$ replaced by $\overline{P}_{rt,u}$, where $\overline{P}_{rt,u} = \mathbb{E}[P_{rt,u}(x, y)]$.

### B. Proof of Theorem 3

In CSMA$_{TX}$, outage occurs if one or more of the following events occur;

*a*) The packet is backed off $M$ times and thus dropped.

*b*) Once the packet transmission is initiated, one or both of the following subevents occur $N + 1$ times:

- $b_1$) SINR$< \beta$ at the start of the packet, i.e., $t = 0$.
- $b_2$) SINR$< \beta$ at some $t \in (0, T)$.

Mathematically, the outage probability of CSMA$_{TX}$ may be expressed as

$$P_{out}(\text{CSMA}_{TX}) = \Pr\left[\text{SINR} < \beta \text{ at } t = 0\right]^M + \left(1 - \Pr\left[\text{SINR} < \beta \text{ at } t = 0\right]^M\right) \times \Pr\left[\text{SINR} < \beta \text{ at some } t \in [0, T)|\text{active}\right]^{N+1}. \quad (17)$$

In the derivation of the probability of these events, we only include the contribution of the nearest interferer, making our formulas only approximate expressions for the actual outage probability. The probability of event $a$) is determined by the number of interferers inside $\mathcal{D} \cap B(\text{TX}_0, s)$ upon the arrival of the packet. This depends on TX$_0$'s own position as well as on the positions of all the other nodes in a chain fashion. The density of packets attempting to access the channel is derived as follows:

$$\lambda_{csma}^{TX} = \begin{cases} \lambda \sum_{m=0}^{M-1} \overline{P}_b^m & ; \text{for } N = 0 \\ \lambda \left[\sum_{m=0}^{M-1} \overline{P}_b^m + (1 - \overline{P}_b^M)\overline{P}_{rt1} \sum_{n=0}^{N-1} \overline{P}_{rt}^n\right] & ; \text{for } N \geq 1 \end{cases} = \lambda \left[\frac{1 - \overline{P}_b^M}{1 - \overline{P}_b} + (1 - \overline{P}_b^M)\overline{P}_{rt1} \frac{1 - \overline{P}_{rt}^N}{1 - \overline{P}_{rt}}\right]. \quad (18)$$

On the other hand, since retransmitted packets do not perform new channel sensing, the density of active packets is given by Eq. (9). Only the first term of Eq. (12) is multiplied by $(1 - P_b)$, because once a transmitter-receiver pair has decided to transmit, it will not perform new sensing and make a new decision at every retransmission attempt. The backoff probability of a transmitter located at $(x, y)$ is then given as: $P_b(x, y) = 1 - e^{-\lambda_{active} A(x,y)}$, where $A(x, y)$ is the area of intersection $\mathcal{D} \cap B(\text{TX}_0, s)$, implicitly dependent on the $(x, y)$ position of TX$_0$.

For the first transmission attempt, $\Pr(b_1|a)$ is found geometrically as the ratio of the area of $B_2 = B(\text{RX}_0, s) \cap \overline{B(\text{TX}_0, s)}$ over the area of $B(\text{RX}_0, s)$, derived to be Eq. (11). For all

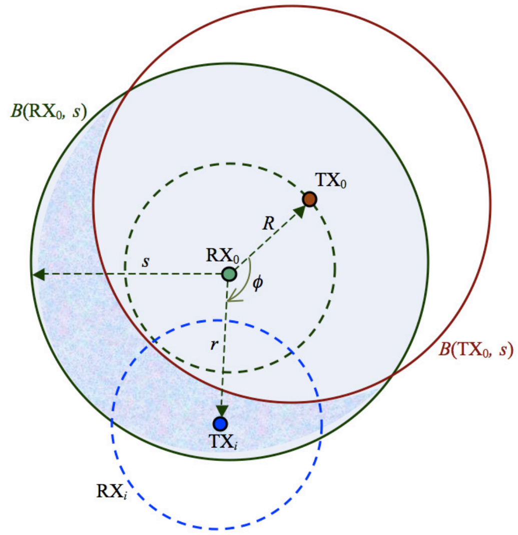


Fig. 6. Setup for derivation of the OP for CSMA.

retransmissions, $\Pr(b_1|a) = \Pr(b_1) = P_b$. The latter equality is because of our assumption that $\beta_b = \beta$.[3].

$\Pr(b_2)$ is lower bounded by the probability that one or more interfering transmitters are located and activated inside $B(\text{RX}_0, s)$ at some $t \in (0, T)$. The integration limits specify the region where the occurrence of an interferer can cause outage for an ongoing transmission, as shown by shaded area in Fig. 6. Demanding that the interferer, TX$_i$, must be at least a distance $s$ away from RX$_0$ in order not to cause outage during transmission, we need to decondition with respect to the position of TX$_i$ to obtain Eq. (10). This requires an averaging over the angle (that we denote as $\Phi$) whose PDF is $1/2\pi$, and over the radius $r$. The limits for $\Phi$ are derived using the cosine-rule:

$$s^2 = r^2 + R^2 - 2Rr\cos(\nu) \quad \Rightarrow \quad \gamma(r) = \cos^{-1}\left(\frac{r^2+R^2-s^2}{2Rr}\right).$$

Integrating $\Phi$ from $\gamma(r)$ to $2\pi - \gamma$, yields $\left[1 - \frac{1}{\pi}\cos^{-1}\left(\frac{r^2+R^2-s^2}{2Rr}\right)\right]$.

Next, consider an angular sector of amplitude $\psi$ radiants, with a node placed on the vertex. Assuming a PPP on the infinite plane, the PDF of the distance to the nearest neighbor within the angle $\psi$, $r$, is given by

$$f_r(r; \psi) = \lambda_{csma}^{TX}\, r\, \psi\, e^{-\frac{\lambda_{csma}^{TX}}{2}\psi r^2}. \quad (19)$$

In a bounded square region, considering $\psi = 2\theta_i(r, x, y)$ (owing to the definition of $\theta_i(r, x, y)$ as a semi-angle defining the $i$-th subregion of the square domain) the same distribution becomes

$$f_r(r; x, y) = \begin{cases} 2\lambda_{csma}^{TX}\, r\, \theta_i(r)\, e^{-\lambda_{csma}^{TX}\theta_i(r,x,y)\, r^2} & ; r_{1,i}(x, y) \leq r \leq r_{2,i}(x, y) \\ 0 & ; \text{otherwise} \end{cases} \quad (20)$$

with $r_{1,i}(x, y)$, $r_{2,i}(x, y)$, and $\theta_i(r, x, y)$ given for $i = 1, \ldots, 8$ in Tables I and II, respectively. Furthermore,

[3]For results on having different sensing thresholds, please refer to [5].

the distribution of $r^2$ is obtained to be $f_{r^2}^{(i)}(r^2; x, y) = \lambda_{csma}^{TX}\, \theta_i(r^2)\, e^{-\lambda_{csma}^{TX}\, \theta_i(r^2,x,y)\, r^2}$ for $r_i \leq r \leq r_{i+1}$.

For more details on the geometric considerations for deriving $P_{during}^{TX}(x, y)$ and $\overline{P}_{rx|active}$, please refer to [5], where an infinite network is assumed. The derived expressions at a given $(x, y)$-coordinate must then be averaged over the entire domain $\mathcal{D}$, as specified in Theorem 3. Inserting the derived expressions back into Eq. (7), we obtain the outage probability of $\text{CSMA}_{\text{TX}}$.

### C. Proof of Theorem 4

The proof of Theorem 4 is similar to that of Theorem 3. The only difference is that $\overline{P}_{rx|active} = 0$, resulting in $\overline{P}_{rt1} = \overline{P}_{during}$. The reason for this is that once the receiver decides to transmit, it is sure not to be in outage at the start of its first transmission attempt. For all other transmission attempts, we have that $\Pr(b_1) = P_b$, because the sensing threshold is equal to the communication threshold, $\beta_b = \beta$.

$\Pr(b_2) = P_{during}^{RX}$ is derived by considering all packet arrivals that arrive during $(0, T)$, are located inside $B(\text{RX}_0, s)$, and are activated. The distribution of the interferer locations is given by Eq. (20). The probability that the receiver of a potential interferer, $\text{RX}_i$, decides to back off, is given by $P(\text{active}|r, \phi)$. This is in effect the probability that $\text{RX}_i$ is placed in $\overline{B(\text{TX}_0, s)}$. Using the cosine rule as explained in Appendix B, we derive the angle of the location where $\text{RX}_i$ can be located as $\theta = \cos^{-1}\left(\frac{d^2+R^2-s^2}{2Rd}\right)$, where $d = \sqrt{r^2 + R^2 - 2Rr\cos\phi}$. The probability that the interfering node is activated is then $\frac{2\pi-2\theta}{2\pi}$. Inserting the equations for $\theta$ and $x$ yields $P(\text{active}|r, \phi)$, as given in Eq. (15).

Inserting the derived expressions into Eq. (13), we obtain the total outage probability of $\text{CSMA}_{\text{RX}}$.

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