Crystallized Rate Regions in MIMO Communications with the Presence of Femtocell Interferers

Adrian Kliks, Paweł Sroka, and Mérouane Debbah

***Abstract*—In this paper the application of the concept of crystallized rate regions to MIMO transmission with femtocell interference is investigated. Based on the referenced work a new game definition specifying the new cost function derived for MIMO transmission with femtocell interference is proposed. Furthermore, the idea of channel approximation for complexity reduction is described. Finally, the simulation results for different MIMO realizations are presented, which prove the correctness of application of the proposed regret-matching algorithm in the MIMO channel scenario with femtocell.**

***Index Terms*—Femtocell Interferer, MIMO, Crystallized Rate Region, regret-matching algorithm.**

I. INTRODUCTION

THE future wireless systems are characterized by decreasing range of the transmitters . The decreasing cell sizes combined with the increasing number of users within a cell greatly increases the impact of interference on the overall system performance. Hence, mitigation of the interference between transmit-receive pairs is of great importance in order to improve the achievable data rates.

The Multiple Input Multiple Output (MIMO) technology in modern communication systems has become an enabler for increase in system throughput. The utilization of spatial diversity thanks to MIMO technology opens new possibilities of spatial resources reuse [1]–[3]. However, as higher transmit frequencies are to be utilized, the range of transmitter decreases, which combined with the increasing number of users within a cell, greatly increases the impact of interference on the overall system performance.

Several concepts of interference mitigation have been proposed, such as the successive interference cancellation or the treatment of interference as additive noise, which are applicable to different scenarios [4]–[6]. When treating the interference as noise the n-user achievable rates region has been found to be the convex hull of n hyper-surfaces [7]. A novel strategy to represent this rate region in the n-dimensional space, by having only on/off power control has been proposed in [8]. A crystallized rate region is obtained by forming a convex hull by time-sharing between $2^n - 1$ corner points within the rate region [8].

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Different approaches are considered when solving the power (or generally resource allocation) optimization problem. Game-theoretic techniques based on the utility maximization problem have received a significant interest recently [7]–[10]. The game-theoretical solutions attempt to find equilibria, where each player of the game adopts a strategy that he/she is unlikely to change. The best known and commonly used equilibrium is the Nash equilibrium [11]. However, the Nash equilibrium investigates only the individual payoff, that may not be efficient from the system point of view. Better performance can be achieved using the correlated equilibrium [12], in which each user considers the others' behaviors to explore mutual benefits. In order to find a correlated equilibrium one can formulate the linear programming problem and solve it using one of the known techniques, such as the Simplex Algorithm [13]. However, in case of MIMO systems the number of available game strategies is high and the linear programming solution becomes very complicated. Thus, a distributed solution can be applied, such as the regret matching learning algorithm proposed in [8], to achieve the correlated equilibrium at lower computational cost. Moreover, the overall system performance may be further improved by an efficient mechanism design, which defines the game rules [14]. In this paper the rate region for the MIMO channel with femtocell interference is examined based on the approach presented in [8], [15]. Specific MIMO techniques have been taken into account such as transmit selection diversity and SVD-MIMO [16], [17]. Moreover, an application of the correlated equilibrium concept to the rate region problem in the considered scenario is presented. Furthermore, a new Vickey-Clarke-Groves (VCG) auction utility [11] formulation and the modified regret-matching learning algorithm are proposed to demonstrate the application of the considered concept for a 2-user MIMO channel with femtocell interference.

The reminder of this paper is structured as follows: Section II presents the considered system model. In Section III the concept of crystallized rates region for MIMO transmission is formulated. Section IV describes the application of correlated equilibrium concept in the rate region formulation. The following Section V outlines the mechanism design for application of the proposed concept in 2-user interference MIMO channel, including the VCG auction utility formulation. Finally, Section VI summarizes the simulation results obtained for the considered specific cases, and Section VII draws the conclusions.

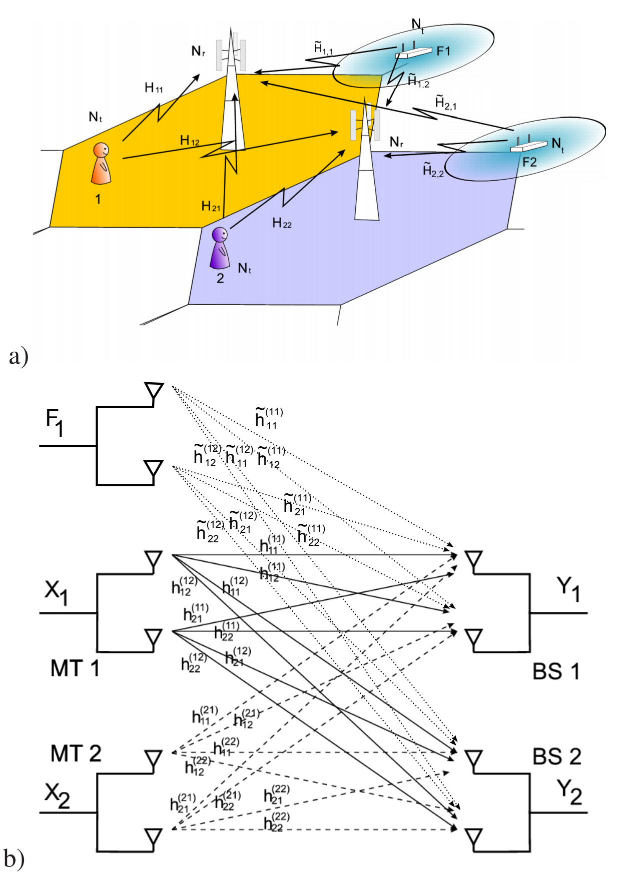

Fig. 1. MIMO interference channel: general 2-cell 2-user model with interfering femtocell AP

II. System model for 2-user Interference MIMO channel

The multi-cell, uplink interference MIMO channel has been considered in this paper, with the 2-cell scenario assumed, in which each of the two users is equipped with the Mobile Terminal (MT) and communicates with his/her own Base Station (BS) causing interference to the neighboring cell. Moreover, presence of femtocell Access Points (APs), causing additional interference to the signals transmitted between users and BSs, has been assumed (see Fig. 1 *a*). Each MT has N_t transmit antennas, each BS has N_r receive antennas and user i can transmit data with the maximum total power equal to $P_{i,max}$. The perfect channel knowledge in all MT's has been also assumed. In order to ease the analysis, we limit our derivation to the 2×2 MIMO case (see Fig. 1 *b*), where both the transmitter and the receiver use only two antennas.
User i transmits the signal vector $X_i \in \mathbf{C}^2$ through the multipath channel $\mathbf{H} \in \mathbf{C}^{4\times4}$, where

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{pmatrix}, \qquad \mathbf{H}_{i,j} \in \mathbb{C}^{2\times2}. \tag{1}$$

The channel matrix $\mathbf{H}_{i,j} = \left\{h_{k,l}^{(i,j)} \in \mathbb{C}, 1 \leq k,l \leq 2\right\}$ consists of the actual values of channel coefficients $h_{k,l}^{(i,j)}$, which define the channel between transmit antenna k at the i-th MT and the receive antenna l at the j-th BS. In the considered 2-user 2×2 MIMO case, only four channel matrices are defined, i.e. $\mathbf{H}_{11}$, $\mathbf{H}_{22}$ (which describe channel between the first MT and first BS or second MT and second BS, respectively), $\mathbf{H}_{12}$ and $\mathbf{H}_{21}$ (which describe the interference channel between first MT and second BS and between second MT and the first BS, respectively). Additive White Gaussian Noise (AWGN) of zero mean and variance σ^2 is added to the received signal. Moreover, interference from each femtocell AP t is added to the received signal, with the interference level depending on the femtocell-to-BSs channel matrices $\tilde{\mathbf{H}}_{t,1}$ and $\tilde{\mathbf{H}}_{t,2}$ for BSs 1 and 2 respectively, with $\tilde{\mathbf{H}}_{t,i} = \left\{\tilde{h}_{k,l}^{(t,i)} \in \mathbb{C}, 1 \leq k,l \leq 2\right\}$. Coefficients $\tilde{h}_{k,l}^{(t,i)}$ define the channel between transmit antenna k at the AP t and the receive antenna l at the i-th BS. Receiver i observes the useful signal, denoted as Y_i, coming from the i-th user. Moreover, in the interference scenario, receiver i (BS$_i$) receives also interfering signals from other users located at the neighboring cell $Y_j, j \neq i$. Interested reader can find solid contribution on the interference channel capacity in the rich literature, e.g. [1], [2], [18], [19]. Following the analysis presented in [18], when interference is treated as noise, the achievable rates for 2-user interference MIMO channel with presence of femtocell AP are defined as follows :

$$\begin{aligned} R_1\left(\mathbf{Q}_1, \mathbf{Q}_2\right) &= log_2(det(\mathbf{I} + \mathbf{H}_{11}\mathbf{Q}_1\mathbf{H}_{11}^* \cdot \\ &\cdot (\sigma^2\mathbf{I} + \sum_t \tilde{\mathbf{H}}_{t,1}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,1}^* + \mathbf{H}_{21}\mathbf{Q}_2\mathbf{H}_{21}^*)^{-1})) \\ R_2\left(\mathbf{Q}_1, \mathbf{Q}_2\right) &= log_2(det(\mathbf{I} + \mathbf{H}_{22}\mathbf{Q}_2\mathbf{H}_{22}^* \cdot \\ &\cdot (\sigma^2\mathbf{I} + \sum_t \tilde{\mathbf{H}}_{t,2}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,2}^* + \mathbf{H}_{12}\mathbf{Q}_1\mathbf{H}_{12}^*)^{-1})). \end{aligned} \tag{2}$$

where R_1 and R_2 denote the rate of the first and second user, respectively, $(\mathbf{A}^*)$ denotes transpose conjugate of matrix $\mathbf{A}$, $det(\mathbf{A})$ is the determinant of matrix $\mathbf{A}$, and $\mathbf{Q}_i$ is the ith user data covariance matrix i.e. $E\left\{X_i X_i^*\right\} = \mathbf{Q_i}$ and $tr(\mathbf{Q}_1) \leq P_{1,max}, tr(\mathbf{Q}_2) \leq P_{2,max}$ with $\tilde{\mathbf{Q}}_t$ denoting the femtocell AP transmitted data covariance matrix. We define the rate region as $\mathfrak{R} = \bigcup\left\{(R_1(\mathbf{Q}_1, \mathbf{Q}_2), R_2(\mathbf{Q}_1, \mathbf{Q}_2))\right\}$.

One can state that the formulas presented above allow us to calculate the rates that can be achieved by the users in the MIMO interference channel scenario in a particular case when no specific MIMO transmission technique is applied. Such approach can be interpreted as the so-called Transmit Selection Diversity (TSD) MIMO technique [16], where the BS can decide between one of the following strategies: to put all of the transmit power to one antenna (N_t strategies, where N_t is the number of antennas), to be silent (one strategy) or to equalize the power among all antennas (one strategy).

When the channel is known at the transmitter, the channel capacity can be optimized by means of some well-known MIMO transmission techniques. Precisely, one can decide for example to linearize (diagonalize) the channel by means of Eigenvalue Decomposition (EvD) or Singular Value Decomposition (SVD) [16], [17]. Such approach will be denoted hereafter as SVD-MIMO.

However, let us stress that (2) has to be modified when the precoding in form of SVD method is applied. Thus, the general equations for the achievable rate computation are defined as follows:

$$\begin{aligned}R_1(\mathbf{Q}_1,\mathbf{Q}_2) &= log_2(det(\mathbf{I}+\mathbf{u}_1^*\mathbf{H}_{11}\mathbf{v}_1\mathbf{Q}_1\mathbf{v}_1^*\mathbf{H}_{11}^*\mathbf{u}_1\cdot\\ &\cdot(\sigma^2\mathbf{u}_1^*\mathbf{u}_1+\sum_t \mathbf{u}_1^*\tilde{\mathbf{H}}_{t,1}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,1}^*\mathbf{u}_1+\\ &+\mathbf{u}_1^*\mathbf{H}_{21}\mathbf{u}_2\mathbf{Q}_2\mathbf{u}_2\mathbf{H}_{21}^*\mathbf{u}_1)^{-1}))\\ R_2(\mathbf{Q}_1,\mathbf{Q}_2) &= log_2(det(\mathbf{I}+\mathbf{u}_2^*\mathbf{H}_{22}\mathbf{v}_2\mathbf{Q}_2\mathbf{v}_2^*\mathbf{H}_{22}^*\mathbf{u}_2\cdot\\ &\cdot(\sigma^2\mathbf{u}_2^*\mathbf{u}_2+\sum_t \mathbf{u}_2^*\tilde{\mathbf{H}}_{t,2}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,2}^*\mathbf{u}_2+\\ &+\mathbf{u}_2^*\mathbf{H}_{12}\mathbf{u}_1\mathbf{Q}_1\mathbf{u}_1\mathbf{H}_{12}^*\mathbf{u}_2)^{-1}))\end{aligned}\tag{3}$$

where $\mathbf{u}_i$ and $\mathbf{v}_i$ denote the set of receive and transmit beamformers, respectively, obtained for the ith user. No precoding has been applied in the femtocell AP with the transmit power distributed equally over the transmit antennas.

One can observe that the complexity of such a solution is relatively high, especially when the number of femtocell users is high. However, what is more important from the perspective of practical implementation of the proposed algorithm is that the characteristic of the transmit channel of the femtocell users could be not known in general. Moreover, even if such a knowledge is available to primary users the amount of backhaul information (that has to be send among all users) grows with the number of femtocell interferers. One way to reduce the number of required mathematical operations and at the same time the amount of backhaul traffic is to quantize the channel. Various approaches regarding the channel quantization can be found in literature. In this paper we have assumed that the base stations have the possibility to approximate the interference from the femtocell users in form of the one value - power of interference expressed as σ_t^2. In such a case the above formulas have to be rewritten as follows:

$$\begin{aligned}R_1(\mathbf{Q}_1,\mathbf{Q}_2) &= log_2(det(\mathbf{I}+\mathbf{u}_1^*\mathbf{H}_{11}\mathbf{v}_1\mathbf{Q}_1\mathbf{v}_1^*\mathbf{H}_{11}^*\mathbf{u}_1\cdot\\ &\cdot(\sigma^2\mathbf{u}_1^*\mathbf{u}_1+\sum_t \sigma_t^2\mathbf{u}_1^*\mathbf{u}_1+\\ &+\mathbf{u}_1^*\mathbf{H}_{21}\mathbf{u}_2\mathbf{Q}_2\mathbf{u}_2\mathbf{H}_{21}^*\mathbf{u}_1)^{-1}))\\ R_2(\mathbf{Q}_1,\mathbf{Q}_2) &= log_2(det(\mathbf{I}+\mathbf{u}_2^*\mathbf{H}_{22}\mathbf{v}_2\mathbf{Q}_2\mathbf{v}_2^*\mathbf{H}_{22}^*\mathbf{u}_2\cdot\\ &\cdot(\sigma^2\mathbf{u}_2^*\mathbf{u}_2+\sum_t \sigma_t^2\mathbf{u}_2^*\mathbf{u}_2+\\ &+\mathbf{u}_2^*\mathbf{H}_{12}\mathbf{u}_1\mathbf{Q}_1\mathbf{u}_1\mathbf{H}_{12}^*\mathbf{u}_2)^{-1}))\end{aligned}\tag{4}$$

Formula (4) can be further simplified by introducing the effective power of interferences $\sigma^2_{\text{eff}}=\sum_t\sigma_t^2+\sigma^2$

$$\begin{aligned}R_1(\mathbf{Q}_1,\mathbf{Q}_2) &= log_2(det(\mathbf{I}+\mathbf{u}_1^*\mathbf{H}_{11}\mathbf{v}_1\mathbf{Q}_1\mathbf{v}_1^*\mathbf{H}_{11}^*\mathbf{u}_1\cdot\\ &\cdot(\sigma^2_{\text{eff}}\mathbf{u}_1^*\mathbf{u}_1+\mathbf{u}_1^*\mathbf{H}_{21}\mathbf{u}_2\mathbf{Q}_2\mathbf{u}_2\mathbf{H}_{21}^*\mathbf{u}_1)^{-1}))\\ R_2(\mathbf{Q}_1,\mathbf{Q}_2) &= log_2(det(\mathbf{I}+\mathbf{u}_2^*\mathbf{H}_{22}\mathbf{v}_2\mathbf{Q}_2\mathbf{v}_2^*\mathbf{H}_{22}^*\mathbf{u}_2\cdot\\ &\cdot(\sigma^2_{\text{eff}}\mathbf{u}_2^*\mathbf{u}_2+\mathbf{u}_2^*\mathbf{H}_{12}\mathbf{u}_1\mathbf{Q}_1\mathbf{u}_1\mathbf{H}_{12}^*\mathbf{u}_2)^{-1}))\end{aligned}\tag{5}$$

III. Crystallized Rate Regions and Time-Sharing Coefficients for the MIMO transmission

The idea of the Crystallized Rate Regions has been introduced in [8] and can be understood as an approximation of the achievable rate regions by the convex time-sharing hull, where the potential curves between characteristic points are replaced by the straight lines connecting these points.

Let us denote each point in the rate region as $\Phi(\mathbf{P}_1,\mathbf{P}_2)$. In such notation, user i transmits with the total power equal to P_i, i.e. $tr(\mathbf{P}_i)=P_i$. Following the approach proposed in [8], [15] we state that instead of power control problem in finding the metrics $\mathbf{P}_i$, the problem becomes finding the appropriate time-sharing coefficients of the $(N_t+2)^n-1$ corner points. For the 2-user 2×2 TSD-MIMO case we will obtain 15 points, i.e. $\mathbf{\Theta}=[\theta_{k,l}]$ for $0\le k,l\le 3\wedge(k,l)\neq(0,0)$, which fulfill the relation: $\sum_{k,l}\theta_{k,l}=1$ [15]. In our case, the time-sharing coefficients relate to the specific corner points, i.e. the coefficient $\theta_{k,l}$ defines the point, where user one chooses the strategy $\alpha_1^{(k)}$ and user two selects the strategy $\alpha_2^{(l)}$. Consequently, (2) can be rewritten as follows:

$$\begin{aligned}R_1(\theta) &= \sum_{k,l}\theta_{k,l}\cdot log_2(\det(\mathbf{I}+\mathbf{H}_{11}\mathbf{Q}_1^{(k)}\mathbf{H}_{11}^*\cdot\\ &\cdot(\sigma^2\mathbf{I}+\sum_t\tilde{\mathbf{H}}_{t,1}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,1}^*+\mathbf{H}_{21}\mathbf{Q}_2^{(l)}\mathbf{H}_{21}^*)^{-1}))\\ R_2(\theta) &= \sum_{k,l}\theta_{k,l}\cdot log_2(\det(\mathbf{I}+\mathbf{H}_{22}\mathbf{Q}_2^{(l)}\mathbf{H}_{22}^*\cdot\\ &\cdot(\sigma^2\mathbf{I}+\sum_t\tilde{\mathbf{H}}_{t,2}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,2}^*+\mathbf{H}_{12}\mathbf{Q}_1^{(k)}\mathbf{H}_{12}^*)^{-1})),\end{aligned}\tag{6}$$

where $\mathbf{Q}_i^{(k)}$ denotes the i-th user covariance matrix while choosing the strategy $\alpha_i^{(k)}$. Let us stress, that any solution point on the crystallized rate border line (frontier) will lie somewhere on the straight lines connecting any of the neighboring characteristic points.

When assuming the quantization of the channel, the formula (6) can be simplified and rewritten as follows:

$$\begin{aligned}R_1(\theta) &= \sum_{k,l}\theta_{k,l}\cdot log_2(\det(\mathbf{I}+\mathbf{H}_{11}\mathbf{Q}_1^{(k)}\mathbf{H}_{11}^*\cdot\\ &\cdot(\sigma^2_{\text{eff}}\mathbf{I}+\mathbf{H}_{21}\mathbf{Q}_2^{(l)}\mathbf{H}_{21}^*)^{-1}))\\ R_2(\theta) &= \sum_{k,l}\theta_{k,l}\cdot log_2(\det(\mathbf{I}+\mathbf{H}_{22}\mathbf{Q}_2^{(l)}\mathbf{H}_{22}^*\cdot\\ &\cdot(\sigma^2_{\text{eff}}\mathbf{I}+\mathbf{H}_{12}\mathbf{Q}_1^{(k)}\mathbf{H}_{12}^*)^{-1})).\end{aligned}\tag{7}$$

Similar conclusions can be drawn for the precoded MIMO systems, where (6), that defines the achievable rate in a time-sharing approach, has to be rewritten in order to include the transmit and receive beamformers set (see (8)).

$$\begin{aligned}R_1(\theta) &= \sum_{k,l}\theta_{k,l}\cdot log_2(\det(\mathbf{I}+\mathbf{u}_1^*\mathbf{H}_{11}\mathbf{v}_1\mathbf{Q}_1^{(k)}\mathbf{v}_1^*\mathbf{H}_{11}^*\mathbf{u}_1\cdot\\ &\cdot(\sigma^2\mathbf{u}_1^*\mathbf{u}_1+\sum_t\mathbf{u}_1^*\tilde{\mathbf{H}}_{t,1}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,1}^*\mathbf{u}_1+\\ &+\mathbf{u}_1^*\mathbf{H}_{21}\mathbf{v}_2\mathbf{Q}_2^{(l)}\mathbf{v}_2^*\mathbf{H}_{21}^*\mathbf{u}_1)^{-1}))\\ R_2(\theta) &= \sum_{k,l}\theta_{k,l}\cdot log_2(\det(\mathbf{I}+\mathbf{u}_2^*\mathbf{H}_{22}\mathbf{v}_2\mathbf{Q}_2^{(l)}\mathbf{v}_2^*\mathbf{H}_{22}^*\mathbf{u}_2\cdot\\ &\cdot(\sigma^2\mathbf{u}_2^*\mathbf{u}_2+\sum_t\mathbf{u}_2^*\tilde{\mathbf{H}}_{t,2}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,2}^*\mathbf{u}_2+\\ &+\mathbf{u}_2^*\mathbf{H}_{12}\mathbf{v}_1\mathbf{Q}_1^{(k)}\mathbf{v}_1^*\mathbf{H}_{12}^*\mathbf{u}_2)^{-1})).\end{aligned}\tag{8}$$

And accordingly, when assuming the quantization of the channel the formula (8) can be simplified and rewritten as follows:

$$\begin{aligned} R_1(\theta) &= \sum_{k,l} \theta_{k,l} \cdot log_2(\det(\mathbf{I} + \mathbf{u}_1^*\mathbf{H}_{11}\mathbf{v}_1\mathbf{Q}_1^{(k)}\mathbf{v}_1^*\mathbf{H}_{11}^*\mathbf{u}_1 \cdot \\ &\cdot (\sigma^2_{\text{eff}}\mathbf{u}_1^*\mathbf{u}_1 + \mathbf{u}_1^*\mathbf{H}_{21}\mathbf{v}_2\mathbf{Q}_2^{(l)}\mathbf{v}_2^*\mathbf{H}_{21}^*\mathbf{u}_1)^{-1})) \\ R_2(\theta) &= \sum_{k,l} \theta_{k,l} \cdot log_2(\det(\mathbf{I} + \mathbf{u}_2^*\mathbf{H}_{22}\mathbf{v}_2\mathbf{Q}_2^{(l)}\mathbf{v}_2^*\mathbf{H}_{22}^*\mathbf{u}_2 \cdot \\ &\cdot (\sigma^2_{\text{eff}}\mathbf{u}_2^*\mathbf{u}_2 + \mathbf{u}_2^*\mathbf{H}_{12}\mathbf{v}_1\mathbf{Q}_1^{(k)}\mathbf{v}_1^*\mathbf{H}_{12}^*\mathbf{u}_2)^{-1})). \end{aligned} \quad (9)$$

IV. Correlated Equilibrium for Crystallized Interference MIMO Channel

In general, each user plays one of $N_s = N_c + 2$ strategies $\alpha^{(k)}, 1 \le k \le N_c$, where N_c is the number of antennas in case of TSD-MIMO and SVD-MIMO ($N_c = N_t$). As a result of playing one of the strategies, the i-th user will receive payoff, denoted hereafter $U_i(\alpha_i^{(k)})$. The aim of each user is to maximize its payoff with or without cooperation with the other users. Such a game leads to the well-known Nash equilibrium strategy α_i^* [20], such that

$$U_i(\alpha_i^*, \alpha_{-i}) \ge U_i(\alpha_i, \alpha_{-i}), \forall i \in S. \quad (10)$$

where α_i represents the possible strategy of the i-th user, whereas α_{-i} defines the set of strategies chosen by other users, i.e. $\alpha_{-i} = \{\alpha_j\}$, $j \ne i$, and S is the users set of the cardinality n. The idea behind the Nash equilibrium is to find the point of the achievable rate region (which is related to the selection of one of the available strategies), from which a user cannot increase its utility (increase the total payoff) without reducing other users' payoffs.

Moreover, in this context, the correlated equilibrium used in [8] instead of Nash equilibrium is defined as α_i^* such that:

$$\sum_{\alpha_{-i} \in \Omega_{-i}} p(\alpha_i^*, \alpha_{-i})[U_i(\alpha_i^*, \alpha_{-i}) - U_i(\alpha_i, \alpha_{-i})] \ge 0, \quad \forall \alpha_i, \alpha_i^* \in \Omega_i, \forall i \in S, \quad (11)$$

where $p(\alpha_i^*, \alpha_{-i})$ is the probability of playing strategy α_i^* in a case when other users select their own strategies $\alpha_j, j \ne i$. Ω_i and Ω_{-i} denote the strategy space of user i and all the users other than i, respectively. The probability distribution p is a joint point mass function of the different combinations of users strategies. As in [8], the inequality in correlated equilibrium definition means that when the recommendation to user i is to choose action α_i^*, then choosing any other action instead of α_i^* cannot result in higher expected payoff for this user. Note that the cardinality of the Ω_{-i} is $(N_c + 2)^{(n-1)}$.

Let us stress out that the time-sharing coefficients $\theta_{k,l}$ are the $(N_c + 2)^{(n-1)}$ point masses that we want to compute. In such a case, the one-to-one mapping function between any time-sharing coefficient $\theta_{k,l}$ and the corresponding point mass function $p\left(\alpha_i^{(k)}, \alpha_j^{(l)}\right)$ of the point $\Phi\left(\alpha_i^{(k)}, \alpha_j^{(l)}\right)$ can be defined as follows:

$$\theta_{k,l} = p\left(\alpha_i^{(k)}, \alpha_j^{(l)}\right), \quad (12)$$

where $p\left(\alpha_i^{(k)}, \alpha_j^{(l)}\right)$ is the probability of user i playing the kth strategy and user j playing the lth strategy.

The problem of finding the above mentioned probabilities can be formulated as the Linear Programming (LP) problem and solved using a well known simplex algorithm [13]. On the other hand, a distributed learning solution may be considered as more cost-effective comparing to the LP [15].

V. Mechanism Design and Learning Algorithm

The rate optimization over the interference channel requires the two major issues to be coped with. First, to ensure the system convergence to the desired point, which can be achieved using an auction utility function. Second, a distributive solution is necessary to achieve the equilibrium.

To resolve a conflict between users, the Vickrey-Clarke-Groves (VCG) auction mechanism design is employed, which aims to maximize the utility U_i, $\forall i$, defined as:

$$U_i \triangleq R_i - \zeta_i, \quad (13)$$

where R_i is the rate of user i, and the cost ζ_i is evaluated as:

$$\zeta_i(\alpha) = \sum_{j \ne i} R_j(\boldsymbol{\alpha}_{-i}) - \sum_{j \ne i} R_j(\alpha_i). \quad (14)$$

Hence, for the considered scenario with two users i and j the payment costs for user i can be defined in (16), where $\mathbf{Q}_i^{(k)}$ and $\mathbf{Q}_j^{(l)}$ are the covariance matrices corresponding to the strategies $\alpha_i^{(k)}$ and $\alpha_j^{(l)}$ selected by user i and user j respectively, that is denoted $\alpha_i \,\widehat{=}\, \mathbf{Q}_i^{(k)}$. The payment cost ζ_j follows by symmetry. Thus, the VCG utilities can be calculated using (15):

$$\{U_i, U_j\} = \{U_i'(\mathbf{Q}_i^{(k)}, \mathbf{Q}_j^{(l)}), U_j'(\mathbf{Q}_i^{(k)}, \mathbf{Q}_j^{(l)})\}, \quad (15)$$

where $U_i'(\mathbf{Q}_i^{(k)}, \mathbf{Q}_j^{(l)})$ is defined as in (17) and (18) for TSD-MIMO and precoded MIMO, respectively. To find the optimal solution of the proposed auction and converge to the correlated equilibrium, the regret-matching learning algorithm proposed in [15] can be applied. Clearly, formulas (17) and (18) can be adopted to the case when the quantization of the femtousers' transmission channel is applied. Such a modification is provided in (19) and (20), respectively.

VI. Simulation Results

A. Case I

To verify the applicability of the proposed distributive approach to the scenario with femtocell interference, the 2-user 2×2 MIMO system has been simulated. The channel matrices considered in simulations case have been set as presented in (21).

$$\begin{aligned} H_{11} &= \begin{pmatrix} 0.9 & 0.00099 \\ 0.00085 & 0.96 \end{pmatrix} \\ H_{22} &= \begin{pmatrix} 0.96 & 0.000096 \\ 0.0000998 & 0.902 \end{pmatrix} \\ H_{12} &= \begin{pmatrix} 0.000094 & 0.00009 \\ 0.992 & 0.9992 \end{pmatrix} \\ H_{21} &= \begin{pmatrix} 0.999 & 0.9904 \\ 0.0005 & 0.0001 \end{pmatrix}. \end{aligned} \quad (21)$$

$$\begin{aligned}\zeta_i\left(\alpha_i \widehat{=} \mathbf{Q}_i^{(k)}, \alpha_j \widehat{=} \mathbf{Q}_j^{(l)}\right) &= R_j\left(\alpha_i \widehat{=} \mathbf{Q}_i^{(0)}, \alpha_j \widehat{=} \mathbf{Q}_j^{(l)}\right) - R_j\left(\alpha_i \widehat{=} \mathbf{Q}_i^{(k)}, \alpha_j \widehat{=} \mathbf{Q}_j^{(l)}\right) = \\ &= log_2\left(\det\left(\mathbf{I} + \left(\mathbf{H}_{jj}\mathbf{Q}_j^{(l)}\mathbf{H}_{jj}^*\right)\cdot\left(\sigma^2\mathbf{I} + \sum_t \tilde{\mathbf{H}}_{t,j}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,j}^*\right)^{-1}\right)\right) - \\ &+ log_2\left(\det\left(\mathbf{I} + \mathbf{H}_{jj}\mathbf{Q}_j^{(l)}\mathbf{H}_{jj}^*\cdot(\sigma^2\mathbf{I} + \sum_t \tilde{\mathbf{H}}_{t,j}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,j}^* + \mathbf{H}_{ij}\mathbf{Q}_i^{(k)}\mathbf{H}_{ij})^{-1}\right)\right),\end{aligned} \tag{16}$$

$$\begin{aligned}U_i'(\mathbf{Q}_i^{(k)}, \mathbf{Q}_j^{(l)}) =& log_2\left(\det\left(\mathbf{I} + \mathbf{H}_{ii}\mathbf{Q}_i^{(k)}\mathbf{H}_{ii}^*\cdot\left(\sigma^2\mathbf{I} + \sum_t \tilde{\mathbf{H}}_{t,i}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,i}^* + \mathbf{H}_{ji}\mathbf{Q}_j^{(l)}\mathbf{H}_{ji}\right)^{-1}\right)\right) - \\ &+ log_2\left(\det\left(\mathbf{I} + \mathbf{H}_{jj}\mathbf{Q}_j^{(l)}\mathbf{H}_{jj}^*\cdot\left(\sigma^2\mathbf{I} + \sum_t \tilde{\mathbf{H}}_{t,j}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,j}^*\right)^{-1}\right)\right) + \\ &+ log_2\left(\det\left(\mathbf{I} + \mathbf{H}_{jj}\mathbf{Q}_j^{(l)}\mathbf{H}_{jj}^*\cdot\left(\sigma^2\mathbf{I} + \sum_t \tilde{\mathbf{H}}_{t,j}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,j}^* + \mathbf{H}_{ij}\mathbf{Q}_i^{(k)}\mathbf{H}_{ij}\right)^{-1}\right)\right)\end{aligned} \tag{17}$$

$$\begin{aligned}U_i'(\mathbf{Q}_i^{(k)}, \mathbf{Q}_j^{(l)}) =& log_2\left(\det\left(\mathbf{I} + \mathbf{u}_i^*\mathbf{H}_{ii}\mathbf{v}_i\mathbf{Q}_i^{(k)}\mathbf{v}_i^*\mathbf{H}_{ii}^*\mathbf{u}_i\cdot\left(\sigma^2\mathbf{u}_i^*\mathbf{u}_i + \sum_t \mathbf{u}_i^*\tilde{\mathbf{H}}_{t,i}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,i}^*\mathbf{u}_i + \mathbf{u}_i^*\mathbf{H}_{ji}\mathbf{v}_j\mathbf{Q}_j^{(l)}\mathbf{v}_j^*\mathbf{H}_{ji}\mathbf{u}_i\right)^{-1}\right)\right) - \\ &+ log_2\left(\det\left(\mathbf{I} + \mathbf{u}_j^*\mathbf{H}_{jj}\mathbf{v}_j\mathbf{Q}_j^{(l)}\mathbf{v}_j^*\mathbf{H}_{jj}^*\mathbf{u}_j\cdot\left(\sigma^2\mathbf{u}_j^*\mathbf{u}_j + \sum_t \mathbf{u}_j^*\tilde{\mathbf{H}}_{t,j}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,j}^*\mathbf{u}_j\right)^{-1}\right)\right) + \\ &+ log_2\left(\det\left(\mathbf{I} + \mathbf{u}_j^*\mathbf{H}_{jj}\mathbf{v}_j\mathbf{Q}_j^{(l)}\mathbf{v}_j^*\mathbf{H}_{jj}^*\mathbf{u}_j\cdot\left(\sigma^2\mathbf{u}_j^*\mathbf{u}_j + \sum_t \mathbf{u}_j^*\tilde{\mathbf{H}}_{t,j}\tilde{\mathbf{Q}}_t\tilde{\mathbf{H}}_{t,j}^*\mathbf{u}_j + \mathbf{u}_j^*\mathbf{H}_{ij}\mathbf{v}_i\mathbf{Q}_i^{(k)}\mathbf{v}_i^*\mathbf{H}_{ij}\mathbf{u}_j\right)^{-1}\right)\right)\end{aligned} \tag{18}$$

$$\begin{aligned}U_i'(\mathbf{Q}_i^{(k)}, \mathbf{Q}_j^{(l)}) =& log_2\left(\det\left(\mathbf{I} + \mathbf{H}_{ii}\mathbf{Q}_i^{(k)}\mathbf{H}_{ii}^*\cdot\left(\sigma^2_{\text{eff}}\mathbf{I} + \mathbf{H}_{ji}\mathbf{Q}_j^{(l)}\mathbf{H}_{ji}\right)^{-1}\right)\right) - \\ &+ log_2\left(\det\left(\mathbf{I} + \mathbf{H}_{jj}\mathbf{Q}_j^{(l)}\mathbf{H}_{jj}^*\cdot\left(\sigma^2_{\text{eff}}\mathbf{I}\right)^{-1}\right)\right) + \\ &+ log_2\left(\det\left(\mathbf{I} + \mathbf{H}_{jj}\mathbf{Q}_j^{(l)}\mathbf{H}_{jj}^*\cdot\left(\sigma^2_{\text{eff}}\mathbf{I} + \mathbf{H}_{ij}\mathbf{Q}_i^{(k)}\mathbf{H}_{ij}\right)^{-1}\right)\right)\end{aligned} \tag{19}$$

$$\begin{aligned}U_i'(\mathbf{Q}_i^{(k)}, \mathbf{Q}_j^{(l)}) =& log_2\left(\det\left(\mathbf{I} + \mathbf{u}_i^*\mathbf{H}_{ii}\mathbf{v}_i\mathbf{Q}_i^{(k)}\mathbf{v}_i^*\mathbf{H}_{ii}^*\mathbf{u}_i\cdot\left(\sigma^2_{\text{eff}}\mathbf{u}_i^*\mathbf{u}_i + \mathbf{u}_i^*\mathbf{H}_{ji}\mathbf{v}_j\mathbf{Q}_j^{(l)}\mathbf{v}_j^*\mathbf{H}_{ji}\mathbf{u}_i\right)^{-1}\right)\right) - \\ &+ log_2\left(\det\left(\mathbf{I} + \mathbf{u}_j^*\mathbf{H}_{jj}\mathbf{v}_j\mathbf{Q}_j^{(l)}\mathbf{v}_j^*\mathbf{H}_{jj}^*\mathbf{u}_j\cdot\left(\sigma^2_{\text{eff}}\mathbf{u}_j^*\mathbf{u}_j\right)^{-1}\right)\right) + \\ &+ log_2\left(\det\left(\mathbf{I} + \mathbf{u}_j^*\mathbf{H}_{jj}\mathbf{v}_j\mathbf{Q}_j^{(l)}\mathbf{v}_j^*\mathbf{H}_{jj}^*\mathbf{u}_j\cdot\left(\sigma^2_{\text{eff}}\mathbf{u}_j^*\mathbf{u}_j + \mathbf{u}_j^*\mathbf{H}_{ij}\mathbf{v}_i\mathbf{Q}_i^{(k)}\mathbf{v}_i^*\mathbf{H}_{ij}\mathbf{u}_j\right)^{-1}\right)\right)\end{aligned} \tag{20}$$

That means that both users have good channel characteristic within their cells (no significant interference exists between the first transmit and the second receive antenna as well as between the second transmit and the first receive antenna). However, the first user causes strong interference on the second receive antenna of the second user, and the second users disturbs significantly the signal received by the first user in his/her first antenna. Although the simulation results are presented only for the above mentioned channel matrix, one should notice that the proposed methodology is valid for every channel case where interference is dominant. For simplicity, only one interfering femtocell AP has been assumed, with the channel matrices for the AP-BSs links specified as:

$$\tilde{H}_{1,1} = \begin{pmatrix} 0.4 & 0.49 \\ 0.52 & 0.46 \end{pmatrix} \quad \tilde{H}_{1,2} = \begin{pmatrix} 0.45 & 0.49 \\ 0.47 & 0.45 \end{pmatrix} \tag{22}$$

Hence, all BSs receive antennas are similarly affected by interference from femtocell AP. The femtocell interference level depends strictly on the selected femtocell AP transmit power value.

Fig. 2 shows the crystallized rate regions for the considered scenario, with four different femtocell interference power to noise ratios: 0 (no interference - region 1), 1 (region 2), 100 (region 3) and 5000 (region 4).

Analyzing the presented results one can observe that the obtained rate regions are concave, thus, the Time-Sharing approach can provide better performance than continuous Power Control scheme (i.e. when both users transmit all the time and regulate the interference level by the means of the value of transmit power). Moreover, the Learned Points, i.e. the points of convergence of the regret-matching algorithm, obtained for TSD-MIMO have been marked in the described figure as LP1-LP4 for the four interference power to noise

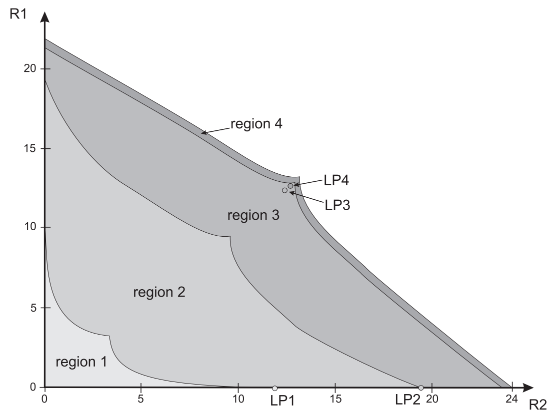

Fig. 2. Rate regions calculated for the considered scenario and the corresponding learned points

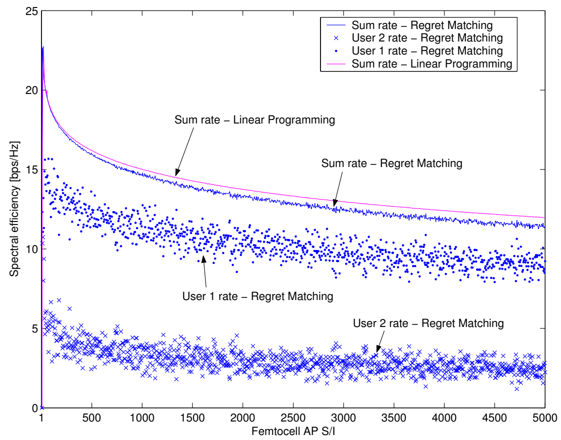

Fig. 3. Achieved rates vs. femtocell AP power to noise ratio in case of TSD-MIMO

ratios, respectively. One can notice, that the proposed auction formulation and regret-matching algorithm provide convergence to the correlated equilibrium, which is the optimal solution of the considered problem.

The influence of femtocell interference power on rates achieved using the proposed distributive solution or the LP approach is shown in Fig. 3 and Fig. 4 for TSD-MIMO and SVD-MIMO, respectively.

One can notice, that the performance of the proposed solution using VCG auction and regret-matching algorithm achieves almost the same performance in terms of sum rate as the more complex LP approach. However, with the increase of femtocell interference power, the gap between these two solutions increases in favor of LP, but the difference in performance is still nearly negligible.

The achieved rates for both TSD-MIMO and SVD-MIMO are similar, thus one can state, that even simple MIMO techniques can be applied in conjunction with the considered algorithms to achieve nearly optimal system performance at lower cost in terms of complexity.

B. Case II

In the second case the deployment of the femto access points was not known, and the characteristic of the transmission channels of every femtocell user have been generated at random fashion. In particular, it has been assumed that the macrocell radius is fixed and equals 500 meters. Moreover, it has been arbitrarily chosen that the femtocell access points are at least 300 meters away from the primary base stations. For both classes of users (primarily, i.e. macrocell users, and secondary, i.e. femtocell users) the pathloss values have been calculated based on the free space transmission model. Furthermore, it has been assumed that the BS transmit power is normalized to unity, and the femtocell access point transmit power P_{femto} was set to 0.1, 0.05 and 0.01. All of the presented results have been averaged over 2000 channel realizations. The obtained results for various values of transmit power of femtocell access point and for various number of femtocells are presented in figures 5 to 7. One can observe that the achievable rate region shrinks when the power of interference increases as the number of femtocell users grows. What is important in each case the learned point indicated by the regret matching algorithm was always the middle point of the frontier line. It means that the algorithm can always select such transmission strategies that both users transmit simultaneously maximizing at the same time the total sum-rate.

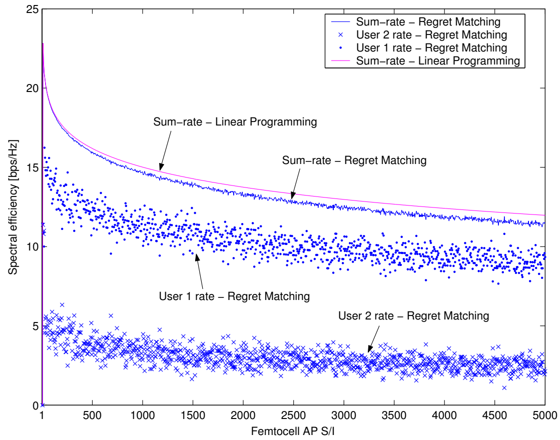

Fig. 4. Achieved rates vs. femtocell AP power to noise ratio in case of SVD-MIMO

In the next step the rates for users i and j, as well the total sum rate, have been investigated for four separate simulation scenarios. Besides the two already introduced cases (TSD- and SVD-based MIMO technique) also the approach with specifically selected beamformers family has been verified. In particular the LTE precoders have been used and the Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) receiver optimization criteria have been utilized for selection of the best precoding vector. In all cases the proposed algorithm have selected a learned point such that the total sum rate is maximized. The obtained results have been compared with the situation when the characteristics of the transmission channel of the femtousers are explicitely known (Case A) and are approximated by the effective interference power paremeter σ^2_{eff} (Case B). The results are presented in table I. One can state that the obtained values in both scenarios (when the channel is known and when the channel is approximated) are similar. It means that the interference coming from the

TABLE I
ACHIEVED RATES FOR BOTH USERS AND THE TOTAL SUM-RATE FOR TWO SIMULATION CASES: WHEN THE FEMTOUSER CHANNELS ARE KNOWN (CASE A) AND WHEN THESE ARE APPROXIMATED BY THE INTERFERENCE POWER (CASE B); N_t - NUMBER OF FEMTOCELLS;

N_t	P_{femto}	Case A			Case B		
		R_1	R_2	Sum rate	R_1	R_2	Sum rate
TSD-MIMO							
1	0.1	12.5807	11.9456	24.5263	12.9103	10.4925	23.4028
	0.05	14.0652	13.5461	27.6113	13.2509	14.6005	27.8514
	0.01	14.6481	14.6506	29.2987	14.6457	14.6484	29.2941
10	0.1	9.6626	9.57	19.2326	9.8956	9.6351	19.5307
	0.05	11.8847	11.7384	23.6231	11.9502	12.3116	24.2618
	0.01	14.6321	14.6462	29.2783	14.6513	14.6463	29.2976
100	0.1	8.2765	8.2173	16.4938	8.3001	8.1734	16.4735
	0.05	10.1835	10.2431	20.4266	10.0844	10.312	20.3964
	0.01	14.423	14.358	28.781	14.4669	14.354	28.8209
SVD-MIMO							
1	0.1	11.2197	10.9865	22.2062	11.1953	11.1928	22.3881
	0.05	13.5016	13.3055	26.8071	14.6447	14.6591	29.3038
	0.01	14.6478	14.6509	29.2987	14.6454	14.6514	29.2968
10	0.1	9.8843	9.7506	19.6349	9.7744	10.0498	19.8242
	0.05	11.9481	12.3101	24.2582	12.5388	11.5353	24.0741
	0.01	14.6424	14.6448	29.2872	14.6439	14.6431	29.287
100	0.1	8.2145	8.2987	16.5132	8.2403	8.2787	16.519
	0.05	10.2282	10.2669	20.4951	10.2856	10.2573	20.5429
	0.01	14.4126	14.3588	28.7714	14.3871	14.3529	28.74
MIMO with LTE codebook; ZF optimization criterion							
1	0.1	11.4828	10.7956	22.2784	11.1984	11.5311	22.7295
	0.05	14.4178	14.6686	29.0864	14.6132	13.1781	27.7913
	0.01	14.6482	14.6523	29.3005	14.6465	14.6431	29.2896
10	0.1	10.0503	9.6995	19.7498	9.6108	10.0083	19.6191
	0.05	12.1658	11.9565	24.1223	11.982	11.9161	23.8981
	0.01	14.6445	14.6516	29.2961	14.6613	14.658	29.3193
100	0.1	8.2257	8.1436	16.3693	8.1693	8.2125	16.3818
	0.05	10.1644	10.2103	20.3747	10.0617	10.1109	20.1726
	0.01	14.3505	14.3696	28.7201	14.4125	14.3546	28.7671
MIMO with LTE codebook; MMSE optimization criterion							
1	0.1	11.3831	11.1498	22.5329	11.3868	11.1502	22.537
	0.05	13.3831	14.6518	28.0349	14.6531	13.8885	28.5416
	0.01	14.6476	14.646	29.2936	14.6468	14.6489	29.2957
10	0.1	10.3768	9.7382	20.115	9.7461	9.5851	19.3312
	0.05	11.6976	11.5486	23.2462	11.8163	12.1034	23.9197
	0.01	14.6344	14.6604	29.2948	14.6611	14.6724	29.3335

femtocell users can be simply interpreted as noise of a certain variance, and the backhaul traffic can be significantly reduced.

VII. CONCLUSIONS

In this paper the idea of crystallized rate regions, introduced first in the context of finding the capacity of the SISO interference channel and later extended to the MIMO interference channels, has been investigated in a scenario with presence of femtocell interference. The idea of exploiting the correlated equilibrium instead of the well-known Nash equilibrium has been verified for the case of 2-user MIMO transmission with femtocell interference. A VCG auction utility function has been derived for the considered MIMO case and the regret-matching learning algorithm has been proposed to solve the formulated game. Simulation results for the selected 2-user scenario with femtocell interference have been presented, which prove the correctness of application of the crystallized rates region to the considered scenario.

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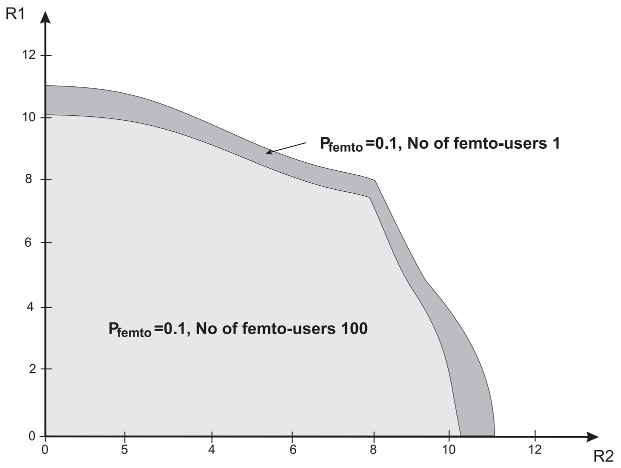

Fig. 5. Achieved rate region for TSD-MIMO for $P_{\text{femto}} = 0.1$; number of femtocells equals 1 (case A) and 100 (case B)

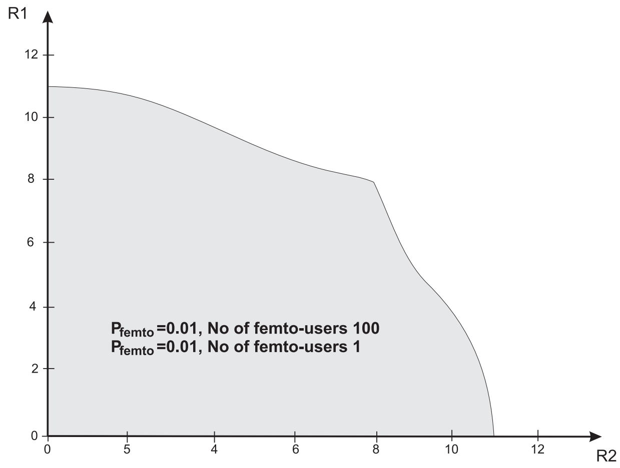

Fig. 7. Achieved rate region for TSD-MIMO for $P_{\text{femto}} = 0.01$; number of femtocells equals 1 (case A) and 100 (case B)

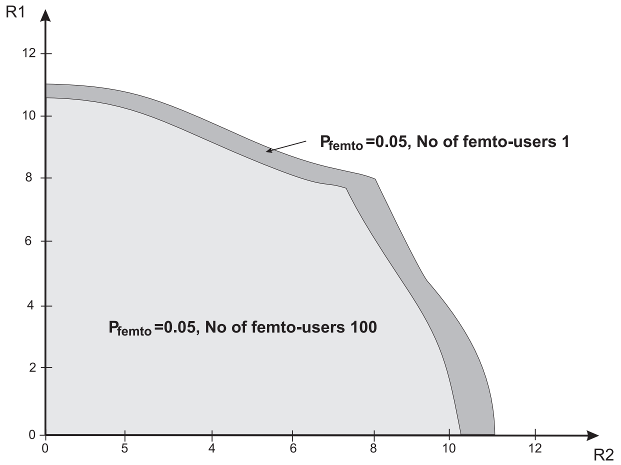

Fig. 6. Achieved rate region for TSD-MIMO for $P_{\text{femto}} = 0.05$; number of femtocells equals 1 (case A) and 100 (case B)

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